*Chapter 1: Indefinite integrals* — *Section 2*

# Terminology and notation

## For indefinite integrals

| *What you need to know already:* | *What you can learn here:* |
| --- | --- |
| ➤ What indefinite integrals are. | ➤ The name and meaning of the different parts of an indefinite integral. |

Indefinite integrals are the fundamental building blocks of integral calculus, so it is not surprising that each detail related to them is important, will be used and deserves a name. We shall start from a very basic, and obvious, piece of terminology.

### Definition

The process of computing the indefinite integral of a function is called ***integration***.

Therefore, ***to integrate a function*** means to compute its general antiderivatives and its indefinite integral.

*Before you move on, I have a question regarding the notation you used in the section on antiderivatives to represent an indefinite integral. What is the strange S-looking symbol that has become so famous to denote integrals?*

It is indeed a strange-looking S! It stands for "*sum*" and to understand why you will have to wait until we introduce the concept of *definite* integrals. If you have already seen them in high school, you may already know what I am referring to, but for now, let us focus some more on indefinite integrals and the process of integration.

However, the main point of this section is to explore the different parts that make up that notation and to give each of them a specific name, since they will be essential in all future sections.

### Definition

When computing an indefinite integral $\int f(x)dx$:

- The symbol $\int$ represents the ***process*** of integration and it is called the ***integral sign***.
- The function $f(x)$ is called the ***integrand***.
- The variable $x$ is called the ***variable of integration***.
- The entity $dx$ is called the ***differential***.

***Example:***

The indefinite integral $\int 3x^2 dx$ represents all functions of the form $y = x^3 + c$. Its integrand is $y = 3x^2$, the variable of integration is $x$ and $dx$ is the differential.

***Example:***

The indefinite integral $\int \cos t \, dt$ represents all functions of the form $y = \sin t + c$. Its integrand is $y = \cos t$, the variable of integration is $t$ and $dt$ is the differential.

These definitions are all that I want you to learn here, and the fact that I have devoted a whole section to this tells you that they are VERY IMPORTANT details. They may look rather simple and insignificant now, but, if ignored, they will lead to severe misunderstandings and problems later. So, here are a few further comments on them.

## Knots on your finger

- The ***integrand*** is the function to be integrated
- The ***variable*** of integration is the independent variable also used to differentiate.

***Example:*** $\int \left(3x^2 + \cos x\right) dx$

Here we are looking for a function $y = f(x)$ such that:

$$f'(x) = \frac{df(x)}{dx} = 3x^2 + \cos x \quad :$$

If, instead, we were given:

$$\int \left(3x^2 + \cos x\right) dt$$

the variable of integration would be $t$ and our procedures for integration would be different according to whether $x$ and $t$ are related or not. If they are not, then the whole integrand would be simply a constant and therefore:

$$\int \left(3x^2 + \cos x\right) dt = \left(3x^2 + \cos x\right) t + c$$

But if they are related, we would need to know what the relationship is before taking any further steps.

This tells you how important it is to pay attention to integrand and variable of integration.

## Knots on your finger

The differential ***dx*** of an indefinite integral:

- is ***related*** to the $dx$ used in the Leibniz notation for derivatives: $y' = \frac{dy}{dx}$
- ***corresponds*** to the differential defined in the context of linear approximations methods
- must be viewed as an ***actual quantity*** that is multiplied by the integrand.

***Example:*** $\int \left(3x^2 + \cos x\right) dx$

In this notation, the integrand $\left(3x^2 + \cos x\right)$ and the differential $dx$ are meant to be multiplied together, and that is why the brackets around the integrand are necessary. If we write:

$$\int 3x^2 + \cos x \, dx$$

we are using an incorrect notation, since the *dx* only multiplies the second term. Does it look pedantic? Maybe, but being precise is at the heart of being successful in calculus.

My last comment also relates to the role of the differential, this time as it relates to a common, but incorrect use of the integral notation.

## *Knots on your finger*

The integral sign must ***ALWAYS be accompanied*** by the differential, in the standard order.

Although the differential has meaning on its own, as we have seen in differential calculus and will see here again, using the ***integral sign by itself is just an amateurish MISTAKE***! Don't do it!

***Example:*** $\int \left(3x^2 + \cos x\right) dx$

The following are incorrect ways to write this indefinite integral:

- $\int \left(3x^2 + \cos x\right)$ , since it does not include the differential.
- $dx \int \left(3x^2 + \cos x\right)$ , since it places the differential to the left of the integral sign.
- $\int 3x^2 + \cos x \, dx$ , since the integral sign is only applied to the first term and the differential multiplies only the second term.

Other errors in notation related to the differential may occur: please avoid them!

## *Summary*

- Each detail of the indefinite integral notation is an important part of it and must be treated correctly and carefully.

## *Common errors to avoid*

- Do not ignore the differential when writing an indefinite integral, since without it the whole thing is meaningless.

# *Learning questions for Section I 1-2*

## *Review questions:*

1. Describe and name all parts of an indefinite integral.

## *Memory questions:*

1. What is the name of function $f(x)$ in the notation $\int f(x)dx$ ?
2. What is the name of the quantity $dx$ in the notation $\int f(x)dx$ ?
3. What is the name of the quantity $x$ in the notation $\int f(x)dx$ ?
4. What is the variable of integration in the integral $\int f(x)dt$ ?

## *Computation questions:*

1. Identify the integrand, the variable of integration and the differential in $\int\left(\ln x - e^x\right)dz$ .
2. Write the indefinite integral that has $f(x) = z^3 - \dfrac{1}{z}$ as integrand.

## *Theory questions:*

1. Explain why the notation $\int x^{-1/2} - \dfrac{2}{x^2}dx$ is incorrect.
2. Is the notation $\int(\sin t - 3t)\,dx$ acceptable?
3. In Leibniz notation for derivatives, the differential $dx$ represents an infinitesimal run. Does it represent the same thing in the notation for integrals?

## ***Templated questions:***

1. In any indefinite integral you find, identify the integrand, variable of integration and differential.

## ***What questions do you have for your instructor?***