### A summary of factoring methods

#### What you need to know already:
- Basic algebra notation and facts.

#### What you can learn here:
- What the word factoring *means*, why it is *useful* and which factoring *methods* are needed for college courses.

Let’s start by clarifying the meaning of the key words related to factoring, words that are used very frequently, but not always in the right way.

**Definition**

A **factor** is a quantity that is **multiplied** with others in a product.

A **term** is a quantity that is **added** with others in a sum.

**Example:** $3x^2(y - 1) + z^3$

In this expression we can see that:
- The quantities $3x^2(y - 1)$ and $z^3$ are terms.
- The quantities $3$, $x^2$ and $y - 1$ are factors of the first term.
- The quantities $y$ and $-1$ are terms of the factor $y - 1$.

Since division is just multiplication by the reciprocal and subtraction is addition with the opposite (you know what these words mean, don’t you?) we can use the same jargon of terms and factors in situations involving divisions and subtractions.

**Example:** $\frac{5 - x^2}{y^3} + xz^2$

In this expression:
- $\frac{5 - x^2}{y^3}$ and $xz^2$ are terms in the whole expression.
- $5$ and $x^2$ are terms in the numerator of the first term.
- $5 - x^2$ and $\frac{1}{y^3}$ are factors in the first term.
- $x$ and $z^2$ are factors in the second term.

*Fine so far, but what does factoring mean as a verb?*
**Definition**

The process of **factoring** an expression consists of changing it from a sum of terms to an equivalent product of factors.

**Example:** \( x^2 - 4 \)

You may remember, and we shall see this soon, that the difference of the two terms \( x^2 \) and 4, is equivalent to a product, as shown here:

\[
x^2 - 4 = (x + 2)(x - 2)
\]

The latter is a product of two factors: \( (x - 2) \) and \( (x + 2) \). So we say that we have factored \( x^2 - 4 \) into \( (x - 2)(x + 2) \).

Isn’t “factoring” just another fancy word for “simplifying”?  

**Warning bells**

Math students (and some teachers and books) tend to use the word simplifying much too often, most times without giving it a specific meaning. The result is that they often identify this word with any manipulation of a formula, often forgetting the reason for such a manipulation.

On the other hand, factoring refers to a specific type of change done with a very clear purpose in mind.

In some situations factoring does simplify an expression, in the sense that sometimes it makes the expression easier to use in the solution of a problem, and in others it makes it shorter. But in other situations factoring makes the expression more complicated or longer, or less useful.

Therefore, it is always a good idea to decide what you want to do with a mathematical expression, and why, before plunging into any manipulation at all, be it factoring or anything else.

**So, when is factoring useful?**

The main situation in which factoring is useful is in the process of solving equations and inequalities. Factoring is an essential step in such process and, therefore, it pops up in all of mathematics. The sections on solving equations and inequalities provide more details on how this is achieved and I strongly recommend that you review them as well.

Is there a process opposite to factoring?

Since in factoring we change a sum to a product, its opposite is the following.

**Definitions**

The process of changing a product of factors to a sum of terms is called **expanding** an expression.

Factoring uses a property of arithmetic called the **distributive** property. This says that for any numbers \( a, b \) and \( c \) the following equality holds:

\[
a(b + c) = ab + ac
\]

The most famous method for expanding, based on the distributive property is the **FOIL** method used to compute the product of two sums:

\[
(a + b)(c + d) = ac + ad + bc + bd
\]

\[F \quad O \quad I \quad L\]
Expanding has its uses, even when solving equations. However, in general factoring tends to be more effective, while expanding is an optional step that only works in some situations. So, use it more sparingly.

So, how do we factor?

There are many ways of factoring an expression, depending on its structure. The rest of this summary presents the basic methods that are used most commonly. Other more advanced methods can be found in specialized textbooks.

Keep in mind that the methods you are about to see are the simplest and most fundamental ones and that you will be expected to know them in any college-level mathematics course.

Here is the very first and most basic one, obtained by reversing the distributive property.

**Strategy for collecting a common factor**

If all terms of an expression have a common factor, this can be written only once and multiplied by the sum of the remaining factors. In formula:

\[ ab_1 + ab_2 + \cdots + ab_n = a(b_1 + b_2 + \cdots + b_n) \]

**Examples:**

- Sometimes the common factor is a constant, such as the number 2 here:
  \[ 2x^2 + 8 = 2(x^2 + 4) \]
- Sometimes it is a variable, such as the \( x \) here:
  \[ x - 3x^2 = x(1 - 3x) \]
- Sometimes it is a power of the variable:
  \[ x^3 - 3x^2 = x^2(x - 3) \]

Sometimes it is a common exponential:

\[
\begin{align*}
2^x + b2^x &= 2^x(1 + b) \\
-ax^3 + ax^{3+2} &= a(-x^3 - a^2x^2) = a(-x^3(1 - a^2))
\end{align*}
\]

Or it may even be a common function:

\[
\begin{align*}
\sin x + \sin^2 x &= \sin(x + \sin x) \\
\sqrt{x+3} + x + 3 &= \sqrt{x+3}(1 + \sqrt{x+3})
\end{align*}
\]

One of the examples above deserves more attention.

When all terms of an expression contain powers of the same quantity, always factor the one with the lowest exponent. In this way, the expression left in brackets contains only constants and powers with positive exponents, a feature that is useful in most cases.

Moreover, with this choice the exponents that are left in bracket are obtained by simply subtracting the chosen common exponent:

\[ ax^p + bx^q + cx^r = x^p(a + bx^{q-p} + cx^{r-p}) \]

**Example:** \( x^3 - 3x^4 + 2x^5 \)

All three terms here include a power of \( x \), with the lowest power being \( x^3 \). We can collect it as a common factor:

\[
x^3 - 3x^4 + 2x^5 = x^3(1 - 3x + 2x^2)
\]
**Warning bells**

This method works even when the exponents are **fractional** or **negative**. In such cases be careful with the arithmetic steps they require.

**Examples:**

We can use this method with positive integer powers:
\[x^3 - 3x^4 + 2x^5 = x^3 \left( x^{-3} - 3x^{-1} + 2x^{-2} \right) = x^3 \left( 1 - 3x + 2x^2 \right)\]

We can use it with negative integer powers:
\[x^{-2} + x^{-3} - 3x = x^{-2} \left( 1 + x^{-1} - x^{-1} - 3x \right) = x^{-2} \left( 1 + x - 3x \right)\]

With fractional powers:
\[2x^{1/3} - x^{2/3} + 5x^{4/3} = x^{1/3} \left( 2 - x^{2/3} - 3x^{4/3} \right) = x^{1/3} \left( 2 - x^{1/3} + 5x \right)\]

And even with negative fractional powers:
\[2^{-1/3} - a2^{-2/3} + a2^{4/3} = 2^{-2/3} \left( 2^{1/3} - x^{2/3} - a + a2^{4/3} \right) = 2^{-2/3} \left( 2^{1/3} - 2a + a^2 2^{2/3} \right)\]

We can use this type of factoring in other, seemingly different situations, by properly interpreting the meaning of negative and fractional exponents.

**Examples:**

\[
\frac{3}{x^3} + \frac{b}{x} - x
\]

Since negative exponents indicate reciprocals, we can use them to obtain common denominators:
\[
\frac{3}{x^3} + \frac{b}{x} - x = 3x^{-3} + bx^{-1} - x = x^{-3} \left( 3 + bx^2 - x^4 \right) = \frac{3 + bx^2 - x^4}{x^3}
\]

**Examples:**

\[
\sqrt[5]{x^3} - 3\sqrt[5]{x} + \frac{2}{x}
\]

Since fractional exponents represent roots, we can use them to group roots:
\[
\sqrt[5]{x^3} - 3\sqrt[5]{x} + \frac{2}{x} = x^{3/5} - 3x^{1/5} + 2x^{-1}
\]
\[
= x^{-1} \left( x^{3/5} - 3x^{6/5} + 2 \right)
\]

**Examples:**

\[
\sec x - \cos x
\]

Since certain trigonometric functions are reciprocal of others, we can use factoring to manipulate and change expressions that contain them:
\[
\sec x - \cos x = \frac{1}{\cos x} - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}
\]

**Examples:**

\[x^4 - 2\]

Sometimes we can collect a common factor even when none is visible! Here there is no common factor, but, for some reason, we may need to factor the power of \(x\). We follow our strategy:
\[x^4 - 2 = x^4 \left( 1 - 2x^4 \right) = x^4 \left( 1 - \frac{2}{x^4} \right)\]

**What about other situations where there are no visible common factors?**

In that case we try to apply the salami technique, by trying to collect factors in the terms that do have a common one.
Strategy for factoring by grouping

When not all terms of an expression have a common factor, but groups of them do, collecting the common factor from each group can still be useful, either by itself or because it may reveal another common factor. This method is called factoring by grouping.

Examples: \(2x^3 + 4x^2 − 5x − 10\)

The terms of this expression do not have anything common to all of them, but the first two terms have a \(2x^2\) in common, while the last two have a -5 in common. Therefore we collect the common factor in each group:

\[
2x^3 + 4x^2 − 5x − 10 = 2x^2(x + 2) − 5(x + 2)
\]

And we now see a common factor between the two new terms, so we collect that one:

\[
2x^2(x + 2) − 5(x + 2) = (x + 2)(2x^2 − 5)
\]

Example: \(x^3x^3 + 3x^3 + 6x + 18\)

We use the same method to first collect a common factor between the first two and the last two terms:

\[
x^3x^3 + 3x^3 + 6x + 18 = 3x^3(x − 3) − 6(x − 3)
\]

Then we collect the newly discovered common factor.

\[
x^3(x − 3) − 6(x − 3) = (x − 3)(3x^3 − 6)
\]

Sometimes you may need to reorganize one of the terms to see the grouping.

Example: \(2x^2 + x − 3\)

The expression does not show any common factors and no clear grouping. We can try to re-write the expression so that a grouping option may appear:

\[
2x^2 + x − 3 = 2x^2 − 2x + 3x − 3
\]

Now we can group as before:

\[
2x^2 + 3x − 2x − 3 = x(2x + 3) − (2x + 3) = (2x + 3)(x − 1)
\]

This looks like a cooked-up example.

It is, and you can use this method only if the terms were set up in such a way as to make it work. Some methods, like collecting common factors, work every time you can use. Some others, like grouping, work only if the terms fit together appropriately.

If you are looking for methods that work every time they apply, you can use special products.

What are they?

Over the years mathematicians have discovered some factoring formulae that can be useful, but require several steps to be proved (usually easy but tedious steps). These formulae are known as special products and the following table lists the ones that are most commonly used, together with their names. In these formulae, \(a\) and \(b\) represent any quantities, be they constant, variables or functions.
**Special product formulae**

<table>
<thead>
<tr>
<th>Formula</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference of squares</td>
<td>$a^2 - b^2 = (a - b)(a + b)$</td>
</tr>
<tr>
<td>Difference of cubes</td>
<td>$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$</td>
</tr>
<tr>
<td>Sum of cubes</td>
<td>$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$</td>
</tr>
<tr>
<td>Difference of fourth powers</td>
<td>$a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$</td>
</tr>
<tr>
<td>Square of a sum</td>
<td>$a^2 + 2ab + b^2 = (a + b)^2$</td>
</tr>
<tr>
<td>Square of a difference</td>
<td>$a^2 - 2ab + b^2 = (a - b)^2$</td>
</tr>
</tbody>
</table>

**Examples:**

For difference of squares:

$x^2 - 4 = x^2 - 2^2 = (x - 2)(x + 2)$

For difference of cubes:

$2^{3x} - 8 = (2^x)^3 - 2^3 = (2^x - 2)(2^{2x} + 2 	imes 2^x + 4)$

For difference of fourth powers:

$\sin^4 \alpha - \cos^4 \beta = (\sin^2 \alpha - \cos^2 \beta)(\sin \alpha + \cos \beta)$

$= (\sin \alpha - \cos \beta)(\sin \alpha + \cos \beta)(\sin^2 \alpha + \cos^2 \beta)$

You can use one of these formulae whenever the expression on which you are working exhibits the same structure as the left side of the formula requires. However, avoid the trap of applying a special product in a situation where it does not apply. For instance in this table there is no formula for a sum of squares. This is because there is no such formula! Making up a wrong formula without checking if it is true is very wrong and will lead to errors.

These look again like cooked up examples

And again, they are! Don’t forget that I design examples and test questions so as to focus on the method, without being bogged down by tedious calculations. But sometimes you may be asked some puzzling questions that still require the application of these formulae.

**Example: $x - y$**

In most cases, there is no need to factor this expression: why would you want to? Well, maybe this is part of a larger expression that also involves the square roots of $x$ and $y$. In that case we can factor it in this strange way by using the difference of squares formula:

$x - y = (\sqrt{x})^2 - (\sqrt{y})^2 = (\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y})$

Of course, this only makes sense if both $x$ and $y$ are positive.
**Example:** 9\(x^2 + 24x + 16\)

Here the first and last terms are perfect squares, so, is this the square of a sum? If we rearrange the expression as:

\[
9x^2 + 24x + 16 = (3x)^2 + 2(3x)4 + 4^2
\]

we see that it is, with \(a = 3x, b = 4\). Therefore we have:

\[
9x^2 + 24x + 16 = (3x + 4)^2
\]

There is one more special case to consider, which involves not so much a formula as a process.

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**Strategy for factoring a quadratic expression**

When factoring a quadratic expression of the form \(ax^2 + bx + c\):

1. Find its two roots, \(x_1\) and \(x_2\), by using the quadratic formula:

\[
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

2. If such roots exist, factor the expression as:

\[
ax^2 + bx + c = a(x - x_1)(x - x_2).
\]

3. If there are no real roots, the expression cannot be factored.

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**Example:** 6\(x^2 - x - 2\)

If we use the quadratic formula, we get:

\[
\frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 6 \times (-2)}}{2 \times 6} = \frac{1 \pm \sqrt{1 + 48}}{12} = \frac{1 \pm 7}{12} = \frac{2}{3} \pm \frac{1}{2}
\]

Therefore we can factor the expression as:

\[
6x^2 - x - 2 = 6\left(x - \frac{2}{3}\right)\left(x + \frac{1}{2}\right)
\]

If we want, we can eliminate the fractions by writing:

\[
6\left(x - \frac{2}{3}\right)\left(x + \frac{1}{2}\right) = 2 \times 3 \left(x - \frac{2}{3}\right)\left(x + \frac{1}{2}\right) = 3 \left(x - \frac{2}{3}\right)\left(2x + 1\right) = (3x - 2)(2x + 1)
\]

Is this a useful step? It depends on what we need next. Remember that any manipulation of an expression should always be done with a purpose in mind, not just to play with it.

You may have seen another method of factoring that requires you to find two numbers whose sum and product satisfy certain conditions. If you have and you like it, by all means use it. However, this method relies too much on guessing, trial and error, so I will not dwell on it here.

**What about situations involving fractions?**

In that case you need to remember that a fraction is a division and therefore it can only be combined with additions and subtractions by following proper rules. Here is an easy and safe way to proceed.
Strategy for factoring with fractions
To factor an expression involving fractions:

1. **Combine** the whole expression into a single fraction by using a common denominator;
2. **Factor** numerator and denominator separately by using appropriate methods.
3. **If changing the domain of the expression is not an issue, cancel factors that appear in both numerator and denominator.**

Example:

\[
\frac{10}{x+1} - \frac{5}{x}
\]

To factor this expression, we first change both fractions so that they have a common denominator:

\[
\frac{10}{x+1} - \frac{5}{x} = \frac{10x}{x(x+1)} - \frac{5(x+1)}{x(x+1)}
\]

Then we combine them:

\[
\frac{10x - 5(x+1)}{x(x+1)} = \frac{5x - 5}{x(x+1)}
\]

The denominator is already factored, so we factor the numerator to conclude that:

\[
\frac{10}{x+1} - \frac{5}{x} = \frac{5(x-1)}{x(x+1)}
\]

Example:

\[
\frac{x}{x^2-1} + \frac{2x}{x+1}
\]

We can begin by factoring the denominator of the first fraction as a difference of squares. This will help us get a common denominator:

\[
\frac{x}{x^2-1} + \frac{2x}{x+1} = \frac{x}{(x-1)(x+1)} + \frac{2x}{x+1}
\]

Now we change the second fraction so as to get a common denominator, then combine the two:

\[
\frac{x}{x^2-1} + \frac{2x}{x+1} = \frac{x}{(x-1)(x+1)} + \frac{2x(x-1)}{(x+1)(x-1)} = \frac{x + 2x(x-1)}{(x-1)(x+1)}
\]

We can now factor the numerator, by collecting the common \(x\):

\[
\frac{x}{x^2-1} + \frac{2x}{x+1} = \frac{x(1 + 2x - 2)}{(x-1)(x+1)} = \frac{x(2x-1)}{(x-1)(x+1)}
\]

Although there are situations in which one can use other methods for factoring expressions involving fractions, this two-step method is quite safe and you should use it until you become more proficient with other methods.

**How far should we keep factoring?**

Once an expression has been factored, it may still be possible to factor each of its factors further. If you look at the example where we factored \(x - y\), you will realize that the process of factoring can theoretically go on and on without ever ending. And yet, sometimes you may be asked to factor an expression **completely**.

**What does this mean?**
**Knot on your finger**

When you are asked to factor an expression completely, keep factoring as long as it is practical to do so, but always keeping in mind the goal for which factoring is needed in your problem.

Stop the process when further factoring does not present any advantage or requires extraordinary methods.

---

**Example:**

\[
\frac{x^2 - 3^2}{x^3 - 27}
\]

To factor this expression completely, we begin by rewriting the negative powers:

\[
\frac{x^2 - 3^2}{x^3 - 27} = \frac{1}{x^2 - \frac{9}{x^2}}
\]

We now combine the fractions in the numerator and move the bottom denominator to the side to avoid multiple layers:

\[
\frac{1}{x^2 - \frac{9}{x^2}} = \frac{9 - x^2}{9x^2 - x^2} = \frac{9 - x^2}{9x^2 - x^2} = \frac{9 - x^2}{9x^2 (x^2 - 27)}
\]

We can keep factoring by applying special products to the numerator and denominator:

\[
\frac{9 - x^2}{9x^2 (x^2 - 3^2)} = \frac{(3-x)(x+3)}{9x^2} \cdot \frac{1}{(x-3)(x^2 + 3x + 9)}
\]

---

\[
\frac{-(x-3)(x+3)}{9x^2 (x-3)(x^2 + 3x + 9)}
\]

Can the quadratic expression be factored? The quadratic formula says no (check it!).

At this point we could keep factoring, for instance letting \(x^2 = x \cdot x\), or by writing \(x - 3\) as a difference of squares, but what’s the point? We notice, instead, that there is factor of \(x - 3\) in both numerator and denominator. If changing the domain of the fraction is not a problem, we can conclude our factoring by stating that:

\[
\frac{x^2 - 3^2}{x^3 - 27} = \frac{x+3}{9x^2 (x^2 + 3x + 9)}
\]

BUT don’t you dare cancelling the \(x\) in the numerator with one \(x\) in the denominator. That is a MAJOR error that will greatly annoy your instructor. Can you explain why it is an error?

And that’s it for now. Work on and become familiar with these methods and you will go far enough in most situations that require factoring.
Summary

- Factoring changes sums to product
- Basic factoring methods include common factors, grouping, special products, common denominators and the use of the quadratic formula.

Common errors to avoid

- **DO NOT** make up factoring formulae and methods, unless you can prove that they are correct. Creativity is great, but creating false facts is not!
- There is no sum of squares formula, so **DO NOT** factor any such sums. Factoring $a^2 + b^2$ as anything is **WRONG**, unless you want to use complex numbers, but for now, use real numbers only.

Learning questions for Section P 1-1

**Review questions:**

1. Explain the purpose of the process of factoring.
2. Describe the relationship between factoring and expanding an expression.
3. Describe how collecting a common factor works.
4. Describe how factoring by grouping works.
5. Explain what a special product is.
6. Describe how to factor a quadratic expression by using the quadratic formula.

**Memory questions:**

1. Are factors parts of a sum or of a product?
2. Are terms parts of a sum or of a product?
3. What is the factored version of the expression $a^2 - b^2$?
4. What is the expanded version of the expression $(a + b)(a - b)$?
5. What is the factored version of the expression $a^3 - b^3$?
6. What is the expanded version of the expression \((a-b)(a^2 + ab + b^2)\)?

7. What is the factored version of the expression \(a^3 + b^3\)?

8. What is the expanded version of the expression \((a+b)(a^2 - ab + b^2)\)?

9. What is the factored version of the expression \(a^4 - b^4\)?

10. What is the expanded version of the expression \((a-b)(a+b)(a^2+b^2)\)?

11. What is the factored version of the expression \(a^2 + 2ax + x^2\)?

12. What is the expanded version of the expression \((a+x)^2\)?

13. What is the factored version of the expression \(x^2 - 2ax + a^2\)?

14. What is the expanded version of the expression \((x-a)^2\)?

15. Which factoring method is needed to factor an expression of the form \(ax^3 + ax^2 - bx - b\)?

**Computation questions:**

Factor completely each of the expressions provided in questions 1-46.

<table>
<thead>
<tr>
<th>1. (3x^4 - 6x^2)</th>
<th>8. (2x^5 - 2x^3 - 3x^2 + 3)</th>
<th>15. (2x^3 + 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. (4x^3 - 8x)</td>
<td>9. (3x^3 + 24)</td>
<td>16. (5x^3 + 9)</td>
</tr>
<tr>
<td>3. (x^3 - 2x^2 - 3x)</td>
<td>10. (2x^3 - 54)</td>
<td>17. (2x^{-2} + 5x^{-3} - 10x)</td>
</tr>
<tr>
<td>4. (x^2 - 3x^3 - 4x^2)</td>
<td>11. (5x^2 + 5)</td>
<td>18. (x^{-2} + 3x^{-1} - 4x^2)</td>
</tr>
<tr>
<td>5. (4x(x+1) - 5(x+1))</td>
<td>12. (8x^2 + 25)</td>
<td>19. (x^{-2} + 3 - 4x^2)</td>
</tr>
<tr>
<td>6. (3x(x-2) + 2(x-2)^2)</td>
<td>13. (3x^4 - 48)</td>
<td>20. (3x^{-1} - 12x^{-3})</td>
</tr>
<tr>
<td>7. (x^5 - 5x^3 - 2x^2 + 10)</td>
<td>14. (3x^4 - 48)</td>
<td></td>
</tr>
</tbody>
</table>
21. \(2x + 1 - \frac{6}{x} - \frac{3}{x^2}\)

22. \(2 + \frac{6}{x} - \frac{1}{x^2} - \frac{3}{x^3}\)

23. \(\frac{x}{x-3} - \frac{x}{3}\)

24. \(\frac{x}{x-3} - \frac{2}{x+3}\)

25. \(\frac{1 + \frac{1}{x}}{x+4}\)

26. \(\frac{x^{-1} + 5^{-1}}{x^2 - 25}\)

27. \(\left(\frac{x-7}{x+1}\right)^2 + \left(\frac{7-x}{x-2}\right)^{-1}\)

28. \(\left(\frac{2x+3}{x+3}\right)^{-1} - \left(\frac{2x+3}{x+2}\right)^{-2}\)

29. \(3\sqrt[4]{x} - 12\sqrt[4]{x^9}\)

30. \(2\sqrt{x} + 4\sqrt{x^5}\)

31. \(4x^{-2/3} + 4x^{1/3} + x^{4/3}\)

32. \(x^{4/3} - x^{1/3} - 2x^{-2/3}\)

33. \(x^{-1/5} + 4x^{4/5}\)

34. \(x^{-8/5} + 3x^{-3/5} + 2x^{2/5}\)

35. \(2^x + 4^x\)

36. \(3^x - 3^{-3x}\)

37. \(4^x - 4^{-x}\)

38. \(5^x - 5^{-2x}\)

39. \(x^2 - x^3 2^x\)

40. \(x^2 3^x - x^4 3^{2x}\)

41. \(\sin^2 x - \cos^2 x\)

42. \(\sin^2 x + \sin 2x\)

43. \(21x^{-3/4} - 17x^{1/4} + 2x^{5/4}\)

44. \(2x^{-1/4} - 3x^{3/4} + 4x^{7/4}\)

45. \(2x^6 - 8x^4 - 3x^2 + 12\)

46. \(\left(\frac{n^3 - 8}{n+2}\right)\left(\frac{2n^2 + 8}{n^3 - 4n}\right)\left(\frac{n^3 + 2n^2}{n^3 + 2n^2 + 4n}\right)\)

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**Theory questions:**

1. Is completing the square one of the methods for factoring?

2. Technically, do we factor expressions, equations or both?

3. How are factoring and solving equations related?
**Proof questions:**

1. Prove that the factorization \( a^4 + b^4 = (a^2 + \sqrt{2}ab + b^2)(a^2 - \sqrt{2}ab + b^2) \) is correct and that neither of the two factors on the right side can be factored further.

**Templated questions:**

1. Construct a (relatively simple) algebraic expression and factor it completely.

**What questions do you have for your instructor?**