

The learning process and how to use it to improve your learning

What you need to know already:

- Learning takes time and involves several aspects.
- In college, you are expected to know certain skills both theoretically and practically.
- You are also expected to be able to demonstrate your knowledge and skills in a variety of ways and problems.

What you can learn here:

- How learning progresses through stages.
- What the purpose of each stage is.
- What activities are effective at each stage and which ones are not.

Introduction

You are now enrolled in a College-level mathematics course; that means that you have several years of experience in education and are used to many, if not all, of the study methods that I will mention here. For instance, you know that to study mathematics it is helpful to read the book, take notes during class, practice, ask questions to the instructor, memorize formulae etc.

However, what you may be missing, since it is often missed, is a good organization, a proper implementation and an understanding of the purposes served by these activities. In particular, only a few students start a study session by asking themselves the following questions:

- What **goal** do I want to achieve now?
- What **activities** should I use to achieve this goal?
- What **limitations** and drawbacks are specific to this activity?
- Is this the **best time** to engage in this activity or to aim for this goal?
- Is this activity best done by myself or in **cooperation** with someone else?

Imagine what would happen if you went through a day doing all the things that you need to do in a day, but at the wrong time and/or for the wrong reasons. Imagine sleeping when you feel hungry, eating to clean your hair, watching TV in order to get clean clothes, or reading a book hoping that this will make you lose weight!

It sounds very silly, doesn't it? And yet, this is what many students do when studying: they use activities that do not meet the needs they have at that moment. Clearly, they will be disappointed when their learning, and hence their grades, will not be what they had hoped for.

In everyday life, you try to do the right things at the right times, by deciding whether to eat, drink, sleep or watch TV according to your present needs and the current stage of development of your day. The same should be done when studying: to identify what is the best thing to do at any time, you need to assess what you need and what your current stage of learning is. The purpose of this unit is to enable you to make this assessment properly.

To "study" or to "learn"?

One word that is used very frequently in a college environment is "study" and its relatives. You are a "student", you need "time to study", your friend cannot join you for pizza because he is "studying for a major test", your roommate did not get a good mark because she "had not studied enough", etc. But what does "to study" mean? According to the [Mirriam-Webster online Dictionary](#) the noun "study" may mean:

- 1) "**A state of contemplation**"
Very poetic, but what are we contemplating?
- 2) "**Application of mental faculties to the acquisition of knowledge**"
Nice purpose, but what are, and how do we apply, "mental faculties?"
- 3) "**A building or room devoted to study or literary pursuits**"
To say that a study is a place where you study does not clarify much!
- 4) "**Purpose, intent**"
Very vague!
- 5) "**A branch or department of learning**"
Which branch of learning?

And, of course, there are many other meanings, even more distantly related to what we need to know.

The problem with focussing on "study" is that this word is too vague. While useful in some contexts, it does not tell us specifically what we are supposed to do, when, how and how much. So, for instance, when people tell you that you need to study more, what do they mean? How much is enough? How do you know that you have completed your studying? No wonder many students get impatient with this kind of advice and end up ignoring it altogether!

However, the fifth definition in the previous list gives us a suggestion: let us look at what "to learn" means. The [same dictionary](#) provides the following definition:

***"To gain knowledge, or understanding or skill
by study, instruction or experience"***

Now, that is much better! Look at what this definition tells us:

- We may claim to have learned once we know, or understand or can do something: this gives us a **goal** and a **criterion** to decide if we have achieved learning or not.

- Studying is one way to achieve your real goal, which is **learning**, but is not itself the goal you are seeking.
- Studying is one of **several ways** to achieve learning: this clarifies the fact that several different activities may and should be used to achieve learning.

For all these reasons, my first advice on how to handle your mathematical work is to focus on *learning*, rather than on *studying*. In particular, you should reduce the emphasis on the issue of "how much time did I study?" and focus more on the question of "how much did I learn?".

Learning as a process

So, we have just discovered that *to learn* means to gain knowledge or understanding or a skill. But how do we get to that point of mastery? If I told you that this is done by swallowing a magic pill or by putting on a learning helmet or by any other kind of instantaneous method, you would probably laugh at me, and rightly so. Everybody knows that learning something does not happen in an instant: it takes time. But do you know what happens during that time? How do you happen to go from not knowing something to being an expert at it?

That is what I call the *learning process*: the extended sequence of stages that we go through as we try to learn something. As you can imagine, this process may be different for different people, at different times, and for different topics and skills. Also, it can be an effective and efficient process, or it can be disorganized and ineffective. You may go through it fully aware of its structure and development, or you may drift through it without any control (and still learn something!). Variety is certainly a key characteristic of the learning process, and it is a good idea to learn to use and enjoy variety, since it is *the spice of life*.

But there are certain features of the learning process that are fairly common and that can be isolated and described so as to help us understand and manage the process better. I have organized these features in what I hope is a simple, but reasonable model. It is not a perfect model, but it may help you clarify some aspects of what you experience and how you feel as you are trying to learn a new topic.

The following table summarizes the six stages of this model and describes the events that identify the beginning and end of each stage. In it, by "*topic*" I mean whatever it is that you are trying to learn, be it a piece of theoretical knowledge, an application or a skill.

<i>The stage of...</i>	<i>starts when...</i>	<i>and is completed when...</i>
<i>Introduction</i>	You are not aware that the topic exists.	You are aware of the topic's existence and, perhaps, of a few basic things about it.
<i>Exploration</i>	You decide you want to learn the topic.	You are somewhat familiar with the topic, but there are still some aspects of it that are not clear to you.
<i>Clarification</i>	You decide to get a better understanding of those aspects that are still unclear to you.	You feel that you clearly understand the topic and that you are comfortable dealing with it.
<i>Confirmation</i>	You decide to check your current level of competence.	You are confident that you do know the topic.
<i>Memorization</i>	You decide that you need to commit to memory certain key aspects of the topic.	You can easily recall all the key aspects of the topic that need to be memorized.
<i>Demonstration</i>	You need to convince others that you are competent in that topic.	You convince others and are officially declared to be an expert.

I hope this model sounds reasonable to you, even though you may be unclear about some aspects of it. That is fine, since you are now at the introductory or exploratory stage of your learning about this model, and hence being puzzled by it is normal. In particular you may be wondering how this model can help you organize your activities. If so, you are ready to explore further or to begin the clarification stage. And the rest of this unit is meant to help you do just that.

Some general features of the learning process model

The first thing that you will notice by looking at this model is that learning does not happen instantly, like turning a light switch on, but develops through a progression of stages, each having its own features and purpose. As a consequence, you should not expect to go quickly from ignorance to competence, but rather you should let time and proper activities work their magic, as you build more and more competence with a topic.

In particular, be prepared for the fact that during the intermediate stages, when you have some knowledge but not enough, you will experience doubts, confusion and feelings of inadequacy. You will also make mistakes! These experiences are all

a natural part of the learning process, even though they make us feel uncomfortable and/or stressed. Being unprepared or unwilling to deal with these unpleasant feelings is one of the main reasons why many students give up or decide to settle for low levels of learning and hence low grades. But if you are aware that you will experience them, and are willing to bear with them, it will be much easier for you to see the process through to a positive conclusion.

Another feature to notice is that four of the six stages begin with your decision to move further with the process. One approach that does not help your learning mathematics is to think that you are being swept along by outside forces (instructor, textbook, tests, technical difficulties) that you cannot control. In fact, while each of those factors does influence your learning, the biggest force shaping it is your determination to do it. Therefore, your first action, when going through a stage, is to decide that you are going to do it. And your decision must include an awareness of what you are getting into, of what it will take to complete it successfully, and of the fact that, yes, you can do it, even though it may not be easy.

The next issue to consider is that each stage has a different starting point and a different conclusion, that is, a different goal. This implies that when starting a study session, it is important to realize what learning stage you are going through. Once you have done this, you will be able to identify:

- the specific *goals* that you can achieve within the current session, thus preventing you from expecting too much or too little from your efforts;
- the *activities* that will allow you to complete your current stage and move on to the next one.

I have described this model as if it were divided in compartments: you begin each stage after the previous one has finished. That is a convenient device to describe the process, but in reality, things do not work out so neatly, as there is always some overlap among the stages. At times you may even find it convenient to retrace your steps and revisit an earlier stage, or to peek ahead at the next one. Feel free to do so, but always according to need, not to lack of organization or by giving away your control of the process.

Still on the question of timing, there are no rules for how much time each stage requires. You may need only a few seconds or many days or weeks to complete a stage. And the time requirements change from person to person and from topic to topic. Do not be rigid on this issue and let your reason decide how much time you need to dedicate to a given stage for a given topic. The critical issue is your awareness of where you are at, not how long you should stay there.

Finally, notice that this model suggests that you deal with the six stages in a certain sequence, which may be different from what you are used to. For instance, many student worry about the memorization stage very early, while it should be an issue only late in the process. Conversely, the issue of checking the status of your learning (confirmation) should come well before your test preparation begins.

Making some of these timing changes may be difficult, since it requires you to change your habits, but it may turn out to be extremely useful, if you want to make the most of your time and of your learning opportunities. So, I suggest that you decide (that word once again!) to make those changes despite the difficulties. Remember that changing our habits and our attitudes feels very strange at the beginning, but that those strange feelings are temporary and open the gate to a more productive and satisfying stage of your life.

It is now time to look at each stage in more detail. But before plunging into the next few pages, spend some useful time to identify any and all issues, questions, doubts, concerns and other aspects that have been raised in your mind by what you have read so far.

Trust me: it will be time well spent and a fruitful experience.

Your notes:

The first stage: "Introduction"

It stands to reason that if you want to learn something you must first realize that it exists! And that part of your learning is what I call the *Introduction* stage.

Interestingly, this critical first stage is, to a very large extent, out of your control: you end up discovering that a certain topic exists and that it will be useful for you to learn it. There are several ways in which this can happen:

- a friend, academic advisor or fellow student **tells** you about it;
- your instructor **mentions** it in class;
- you **read** about it in the college calendar, the course outline or a textbook;
- you **notice** it while reading a magazine or newspaper;
- you **hear** about it from a radio or TV program,
- occasionally, it is brought to your attention by your own **experience**, while you are working on something else.

Whatever the source, this is basically something that happens to you, as opposed to your working on it actively. It is a passive stage, the **only** passive stage.

There are, however, several things you can do to put yourself in a situation where you can receive this introduction. For instance, the following activities can lead you to a fruitful introduction to new topics:

- **Attend** all classes.
- **Talk** to people, especially people more knowledgeable than you, or people who are experts in your field.
- **Read** the table of contents of your textbooks and your course outlines at the beginning of a course, and then again, every here and there.
- **Read** the appropriate section of the textbook before the class when that section is discussed.
- **Read** the college calendar, especially the sections related to your program.
- **Read** informative newspapers, magazines, journals etc.

Some students claim that "*They don't really work for me: I don't get much out of them.*" But remember that these activities serve as *introduction*: because of their limited scope, they are not supposed to do much for you! The introductory stage will not make you an expert on the topic, it will only open a door for you and, hopefully, it will make you curious. Therefore it is perfectly fine to feel confused, unclear and unable to do anything with the topic at this stage. All you need is the awareness and the spark of interest to move you to the beginning of the next stage.

But you now have a **decision** to make: do you want to learn the topic you have just discovered? If the topic appeals to you, if it sparks your curiosity, than this

decision is easy to make: yes, you want to learn it. It is also an easy decision if the topic does not interest you and you can continue through life without knowing much about it: you just abandon that topic!

But what if the topic is one that does not appeal to you, but you still need to learn it in order to achieve your ultimate goal? For instance, I have not met many students who were very interested in, or curious about, learning how to factor polynomials. Yet they had to learn that skill in order to pass their math course and be admitted to, or continue in their program of studies. What do you do then?

You may want to ask yourself the following questions:

- If this is one of **many** topics that you are asked to learn, but find uninteresting, could it be that you are involved in a program that does not suit your personal needs, strengths and goals? And, therefore, should you review your overall academic plans?
- If this is one of **few** topics that you find uninteresting, what can you do to get enough motivation to overcome your feelings and get the job done?

It is only by positively resolving these questions that you will be able to proceed to the next stage. Otherwise you may get stuck at this stage; or, in frustration, you may be tempted to skip all the way to the last stage, that is, focus on test preparation. Doing that is an almost sure prescription for disaster!

If you are debating this decision, keep in mind, again, that talking to appropriate experts can be very helpful. Make an appointment with an academic advisor. Or with a study skills instructor. Or with your course instructor. Or with the person responsible for your program area. Or with all of them! Each of these people can give you good ideas and suggestions, but none of them will force you in any particular direction. The final decision still needs to be yours, but you will be able to make an informed decision, and hence one that you will never regret.

Your notes on the introduction stage:

The second stage: "*Exploration*"

By deciding that you want to learn a new topic you have effectively completed the introduction and moved on to the next stage. What now?

To move any further in your learning you now need to actively get involved with the topic by exploring it. That means doing activities that will give you a good perspective on the topic. Here is a partial list of the things you can do:

- **Participate** in class. Don't just *be* there. Get involved:
 - Do what the instructor asks you to do, rather than watching others do it.
 - Answer all the questions the instructor (or somebody else) asks. You don't have to make your answers public, just get them.
- **Try** any exercises you can find on the topic. Don't be concerned with doing them all correctly, but do try many of them and do your best.
- See if you can **reproduce**, on your own, the examples presented in resources. Don't just read them: do them on your own and then compare your work to what the "experts" have done.
- **Edit** the notes you took in class by checking, completing and correcting them.
- Ask "**What if**" questions:
 - What if the numbers were different?
 - What if I could not use the calculator?
 - What if it was a different shape?
- Look for connections:
 - How does this relate to what we studied last time, or last week?
 - How does this connect with everyday life?
 - How will I be able to use this in my profession?

I suspect that you already do most of these activities, but look at the emphasis for this stage: you want to "*do*", or "*try*", or "*see if*", or "*look for*". There is no requirement, at this stage, that your work be fully correct, and that you get all the right answers. Surprised? Then may be it is time to clarify a few common misconceptions about this stage.

First of all, the purpose of this stage is to *explore*, NOT to complete your learning. You will acquire some experience with the topic, but you will not know all of its aspects. You will try to use your new skills and you will start feeling the satisfaction of improvement, but you may also run into obstacles and may experience some feelings of disappointment. This stage is in fact very similar to exploring a new city. You will be happy to discover new places and to meet new people, but you may still get on the wrong bus, or go to a certain store at a time when it is closed, or miss that most beautiful park in the city.

What I am saying is that at this stage you may make mistakes, you may become confused, and you may even experience feelings of incompetence. Unfortunately, these are unpleasant experiences, but they are part and parcel of the learning process and main ingredients for this stage. To become able to deal with them you need to start viewing them in a positive light. Here are some ideas to help you with that .

- **Mistakes:** It has been said that learning opens the doors to your future. Making a mistake is like using the wrong key for the door you want to open. When that happens, you have a choice: you can look at the closed door in frustration and pessimism, or you can try to figure out why the chosen key is wrong and try another. Of course, by choosing the second approach you will also improve your key-choosing skill and will do better in your next task. Therefore mistakes can be incredibly effective learning tools and we should cherish and treasure them, rather than avoiding them. The key here is not in getting the correct answer all the time, but knowing how to make use of your mistakes to learn more. In fact it has also been said that "*he who never made a mistake never made a discovery*". Remember that the only time when you do not want to make mistakes is during a test, but that is the last stage of the process and you are only on the second stage. So, relax, and learn how to befriend mistakes.
- **Confusion:** When we feel confused, we feel uncomfortable. But what is confusion if not the realization that there is some learning still to be done? Some feelings are there to make us enjoy our life experiences. Others are there to send us a warning message, and confusion is one of them. The feeling of confusion is your brain's method to let you know that it is aware of something that it has not mastered yet. Nothing bad about it: just a warning that a new door has opened, but you are not sure of what is on the other side yet. Notice that you cannot feel confused about something that you do not know at all. Therefore feeling confused tells you that you have started your learning process and have taken enough steps to be able to identify what it is that you are not good at yet. That is progress!
- **Incompetence:** Sometimes, at this stage you may feel incompetent and inadequate. Well, if you were competent and adequate, you would not be at this stage, but at the demonstration stage, or further! Being a student (that is, a learner) is all about going from not being competent to being an expert, and therefore the fact that at this early stage you feel incompetent is a natural and healthy consequence of realizing that you are a learner. And remember that we are all learners. Your instructors may know more mathematics than you do, but they are also constantly learning more mathematics and more about how to teach it. And so they also make mistakes and sometimes feel incompetent. Nothing wrong with it, as long as you keep working at becoming competent.

As you can see, there is absolutely nothing wrong with making mistakes and feeling confused or incompetent at this stage. What damages many students is that they try to eliminate these experiences by avoiding the topic! Instead of gathering the courage to explore what is on the other side of the door, they shut the door. And with that they may close the door to achieving their goals and their dreams.

But what if your feelings are so strong that they prevent you from even trying to overcome them and you just can't handle the thought of making a mistake? Some students are in that situation, often as a result of some traumatic past experience. If that is the case, you are facing a psychological block which can be eliminated, but only with the help of a competent professional. Most colleges offer their students counselling services and it is worth your while to make an appointment with one of these friendly people and allow them to help you acquire the peace of mind that you need to achieve your goals.

Another important aspect of the exploration stage is that it is a very personal stage, you could even call it a lonely stage. You cannot explore a new city by watching somebody else's pictures of it, because your impressions of that city may be very different from theirs. You may even see a city differently from somebody who has toured it with you. Similarly, when you are exploring a new mathematical topic you are the only one who can do it. The instructor can give you directions, your class mates can share that experience with you and hence make it more pleasant, anyone who loves you can support you, but you are the only one who can actually DO it! So this is a very active, very personal, very internal stage. Even if you are working with a study buddy or in a study group, you explore *alongside* others, not *with* them. Your experience is going to be different from anyone else's, and hence you need to take the initiative and the responsibility for it. If you let other people take charge of this, you will, in effect, miss it and will become unable to move to the next stage.

And speaking of later stages, here are two more exploration activities you can do in preparation for them:

- Identify any aspects of the topic that you feel are under your control. You will use them two stages down the road.
- Identify any exercises that you could not complete or that you did incorrectly. You will use them in the next stage.

Your notes on the exploration stage:

The third stage: "Clarification"

You have now explored a topic to a good extent, looked at it from several perspectives, completed a number of exercises, made some (or many) mistakes and identified sources of confusion, you are ready to move to the *clarification* stage.

Mistakes and confusion are useful for you, but to use them to your advantage, you must understand the reasons for your mistakes and confusion and you must learn how to eliminate them. So, you must actively decide to go through this stage.

Contrary to the exploration stage, the stage of clarification is a collaborative one. The best way to clarify your doubts is to talk about them with somebody else. For many students, this is not a comfortable thing to do, as they feel reluctant to admit that they need help. As a result, they choose to skip this stage rather than work through it. The fact is that you cannot skip this crucial stage and still hope to obtain good results in an efficient manner. This problem is particularly acute for males, who are often slaves to a macho image that associates need for assistance with weakness. That association is a fake and dangerous one. In reality, involving somebody else at this stage is not an admission on weakness, rather a clear demonstration that you are aware of your needs and your goals and are determined to use all available tools to achieve your goal.

The central activity of this stage is to formulate a question for each mistake you made and for each unclear issue that you have encountered. These questions should

be focussed, clear and to the point. Once you have those questions, you can raise them with many potential supporters: your instructor, other instructors, a friend who has already learned that topic, a class mate, etc.

Most likely, one or more of these people will help you reach that "aha!" moment that you are looking for. But it is also possible that they will not give you sufficient help. Even your instructors may give you a technically correct answer, but one that does not clarify your difficulties. Remember that your exploration was different from theirs and they may not be able to see what you see and hence give you answers that meet your needs. If that happens you may try somebody else or put that issue on the backburner, to be revisited later, once you learn other related aspects of the topic. Remember that the process is not linear!

Or you may want to improve your questioning skills. An instructor may not help enough because you did not ask the right question. For instance, some students state that they "don't understand it at all". That is rarely true, but does not help the instructor. Much better to focus on one aspect of the topic that you are finding difficult and make it clear. Then you will both be working on the same wavelength and on the same assumptions and the results can be much more positive.

You should also avoid a confrontational style of questioning and try, instead, to bring out your questions in a positive, constructive manner. Here is a small table with some suggestions for how to ask for clarification:

<i>Instead of saying ...</i>	<i>Use the approach ...</i>
I don't get any of it!	I am not clear on this particular point.
How do you do this?	What did I do wrong here?
That seems stupid!	What is the rationale for that?
What's wrong with that?	What mistake did I make and what is the correct way to do it?
I did not understand this topic in class: can you please explain it to me again?	I am having some difficulties understanding this topic: what do you suggest that I do to clarify it?
I had to skip class: did I miss anything?	I apologize for skipping class: which topics did you discuss and what can I do to catch up?

The fourth stage: "Confirmation"

Imagine a famous singer reading the score of a new song, going through the song in his mind, then giving a public performance without any rehearsal. You would think that this person was either a very poor professional, or out of his mind, or both! And yet that is exactly what many students do: they wait until they have to perform in a test before they check whether they have learned a topic or not.

Notice that I am not talking about test preparation here: that will be the last stage. I am talking about asking yourself if you have really explored enough, whether you have answered all the questions, corrected all the mistakes and clarified all the doubts (within the limits of your level of knowledge). In other words, are you ready to move towards exams, or do you need to go back to some of the earlier stages because, after all, you did not really complete them?

This is what the confirmation stage is all about: you want to confirm to **yourself** that, by George, you got it! And if you haven't, have enough sense and time to explore and clarify some more.

Here are some activities that you can do to confirm your knowledge:

- Try some exercises you have done before to make sure that you can now sail **easily** through them.
- Do some **additional** exercises, from any available resource, that are similar, but not equal, to the ones you have already done.
- **Check** with your instructor or other students and friends whether the work you have done on an assignment is correct before you submit it.
- **Recite** definitions, formulae and procedures in *your own words*, to make sure that you know them, rather than simply remember them.
- Get together with a class mate to **ask** each other questions and see if you can answer them. You may even rely on a supportive friend or relative to ask you questions from the book: they don't have to know or understand the answer, since your conscience will tell you if you can do it or not.

Your notes on the clarification and confirmation stages:

The fifth stage: "**Memorization**"

Let me make one thing clear right away: our brains are designed so that they forget things. It's genetic. Some people forget more, some forget less; some people forget certain things, other people forget other things. But we all forget. Just think of the stereotype of the absent-minded professor who knows all the mathematics there is, but forgets his children's birthday or some even more embarrassing thing. We all forget, and we must learn how to live with that.

We forget mostly for two reasons:

- Certain memories can do us more harm than good, and so the brain wipes them out in self-defense. This is what happens, for instance, with some traumatic experiences we had that can interfere with our desire to take a positive approach to life.
- We forget things when there is no use for our remembering them. When was the last time you stayed in a hotel? If it was more than a month ago, do you still remember your room number? I am sure that you knew it at the time: it was useful, if not essential. If you don't remember it any more, that is because your brain has realized it does not need that piece of information any more and has thrown it away. If you do remember it, why do you? Since your brain has only a limited capacity (very big, but still limited), wouldn't it be better to get rid of that information and make space for more useful stuff, such as your current shopping list?

Although some mathematical memories may belong to the first category, most forgetting in a mathematics course belongs to the second: you don't use mathematical facts often enough and most likely your life does not depend on them. So your brain considers them non-essential and chucks them. Math teachers remember many more math facts than you do because their salary depends on that!

But another interesting fact about our memory is that when we forget something, that item is not always totally gone from our brains: more often it has been stored in a way or in a location that we cannot access easily. This fact has two interesting consequences:

- It is very important to use learning activities that facilitate proper long-term memorization. So, choose understanding over rote drills, active over passive exercises, organized over loose lecture notes, making connections over working on disjoint topics, etc.
- You don't need to re-learn a topic that you learned earlier, provided you *learned* it, rather than just *saw* it. All you have to do is refresh your memory enough so that you can access it adequately.

If you use learning activities that facilitate long term memorization, when you actually need to memorize items for a specific purpose, you will require a minimum amount of intensive memorization. As long as you are working in a setting that allows you to use your textbook or other written notes, you don't need to memorize too much. After all that is why writing was invented: to allow us to remember things that we may forget. On the other hand, when you are getting closer to a situation where you will need to recall facts stored in your brain only, then memorization starts having an important role and you will need to focus on it.

Too many students are so worried about memorizing facts and formulae for a test they have in a month, that they don't try to actually learn those facts and formulae! The result is that they do not focus on exploring and clarifying, so they do not learn. Small wonder they do poorly on the test. It wasn't memory they lacked, it was knowledge!

Some of the memorization techniques that are frequently suggested to students include:

- **recite** formulae and definitions repeatedly;
- prepare and use **flash cards**;
- organize a **summary sheet** of formulae and/or a glossary of jargon terms;
- hold frequent sessions with your study buddies to **review** the material.

If you are interested in using some of these techniques, talk to a college counsellor or study skills instructor about them. There are also some memory techniques that are offered on a commercial basis and promise wonders. Most of them have a solid rationale and work well, so you may want to try them, if they interest or impress you. But do keep in mind that what will improve your memory is the quality and quantity of effort you put into it, not how much money you pay.

Your notes on the memorization stage:

The sixth stage: "Demonstration"

By this time, you know the topic. In fact, you are an expert in the topic! But, as the Wizard of Oz would say, that is useless, unless you have a diploma to prove it. And to have a diploma you must convince some authority that you have become an expert. In college that means you must pass tests. That is what the demonstration stage is all about: getting ready to convince some else of your competence.

Notice how this stage is different from the others five for two major reasons:

- Its existence is caused by an **external** factor: you haven't really decided that you want to be tested, rather it is something that you accept to endure in order to achieve your goal of being considered an expert.
- You are not trying to learn something you don't know yet, as it was before, you are trying to **convince** others that you do know it.

These two major differences suggest that this stage is substantially different from the others, must be dealt with differently and is deserving of a separate discussion in a separate document. So, while you look for a thorough discussion of effective test-preparation methods, keep in mind that yes, Virginia, the activities that will prepare you for a test are different from those you use in your regular learning work.

A few students actually enjoy this particular stage. They do not see it as an external imposition, but as a welcome challenge to demonstrate their learning and summarize their gained expertise to themselves as much as to the rest of society. That is certainly true. In fact I hope you do reach that state of mind, since it is a major indication of your having become an excellent student, one who can find internal motivation for learning. But even if you are not there yet, preparing for and writing a test does not have to be the experience of torture that some students perceive.

Implementing the ideas of the model

"This was all very interesting, but what can I do with it?"

A very sensible question, since you are reading this in order to improve your learning skills. So, what does this model imply that you should do? Here are some specific suggestions stemming from the model, to help you learn math more efficiently and effectively.

- **Take the initiative and the responsibility** for the learning process: other people can affect that process, but it is your active involvement in it that will lead you to success.
- **Focus on learning, not studying**, that is, focus on the skills or methods that you need to acquire, not on the time devoted to that vaguely defined notion of studying.
- Before you begin a learning session, **identify which stage you are at**, so that your goals for the session are clear.
- Make sure you **go through all of the stages**. Skipping a stage may make any later stage difficult, if not impossible, to complete.
- **Be aware of the feelings** associated with each stage, especially those that are uncomfortable, and get into the habit of using them as indicators of your progress through the stage, rather than as negative experiences to be avoided.
- Remember that **you will need some lonely time and some cooperation time**; and they must come at the right time, that is, during the right stage, to be effective.
- **Don't be afraid of the length of the process**: if you decide to do it, it will be over successfully earlier than you think. Then you can party!

Your final notes:

Summary

- In order to obtain the maximum benefits from your learning activities, it is important to do each activity at the right time and for the right purpose.
- It is much more useful to focus on the idea of learning than of studying, since the former is better defined and leads to a clear recognition of achievement.
- Learning does not happen instantaneously, but proceeds through several stages, which may be identified in a simple model.
- Different people go through the learning process in different ways and experience different feelings. You must adapt the model to your own individual characteristics.
- The *Introduction* stage is mostly passive and only leads to awareness about a topic.
- The *Exploration* stage is very active and personal and often involves mistakes, confusion and feelings of inadequacy.
- The *Clarification* stage is essential, but it requires cooperation with experts and a clear identification of your areas of technical weakness. It also needs an admission of our frequent need for help.
- The *Confirmation* stage is often skipped, but it provides you with very valuable information on the progress of your learning.
- The *Memorization* stage is generally overestimated to the detriment of the more essential aspects of experiencing and understanding a topic.
- The *Demonstration* stage is also known as test preparation and is discussed in depth in other documents.
- The ideas of the model may be used to introduce practical improvements in your study habits.

Common errors to avoid

- Don't fall into the trap of thinking that learning can be achieved in a disorganized way.
- Don't go through learning by being blissfully unaware of what is happening to you.
- Don't be scared by feelings of inadequacy: we are all inadequate when we start learning something. It is only the rare and weird genius who jumps to the end in one step!
- Don't waste time using a learning activity at a time when it is not appropriate. In particular, do not skip ahead to later stages without completing the earlier ones.
- Don't be fooled into believing that understanding how someone else solves a problem means that now you can do it too.
- Don't be fooled into believing that someone who can solve a problem does it out of sheer intelligence: there are usually many hours of practice and many mistakes behind what looks so easy!

Solutions to selected Learning questions on the learning process:

Review questions:

1. Explain why focussing on learning is better than focussing on studying.
2. Explain why it is important to identify the goals of a learning session before you begin it.
3. Describe how learning develops in a typical situation.
4. Discuss what is common in the way different people learn and what is different.
5. Describe the six stages of learning process presented in this model.
6. Identify the uncomfortable feelings that may occur naturally during the learning process and explain why it is important to be aware of them and to use them to your advantage.
7. Separate the stages of the learning process according to whether they are done individually or in cooperation.
8. Explain in what way the six stages of the learning process occur in a sequence, in what ways they may overlap and in what situation a reversal of the sequence may be called for.
9. Discuss the role of time in an effective use of the learning process.
10. Discuss the role that conscious choices play in the successful implementation of the learning model.
11. Describe the main goal of the Introduction stage, some ways in which it is implemented and what is left to achieve after its completion.
12. Describe the main goal of the Exploration stage, some ways in which it is implemented and what is left to achieve after its completion.
13. Describe the main goal of the Clarification stage, some ways in which it is implemented and what is left to achieve after its completion.
14. Describe the main goal of the Confirmation stage, some ways in which it is implemented and what is left to achieve after its completion.
15. Describe the main goal of the Memorization stage, some ways in which it is implemented and what is left to achieve after its completion.
16. Describe the main goal of the Demonstration stage, some ways in which it is implemented and what is left to achieve after its completion.
17. Discuss the role of mistakes in the learning process, especially with respect to when they are expected, when they are necessary and when they are to be avoided.
18. Discuss the role of confusion in the learning process, especially with respect to when it is expected, when it is useful and when it is to be avoided.
19. Discuss the role of feelings of inadequacy in the learning process, especially with respect to when they are expected, when they are useful and when they are to be avoided.
20. Compare effective and ineffective ways to ask questions to someone who can help you through the learning process.
21. Compare advantages and disadvantages of remembering and forgetting.
22. Identify the activities that may be useful for more than one stage of the learning process and describe how the overlap can be exploited.

Self-assessment

1. Which aspect of this unit are you most clear about and how do you know that you are clear about it? Be as specific as you can.
2. Which aspect of this unit are you most unclear about and why? Be as specific as you can.
3. What is the one most important thing you have learned from this unit? Why is it so important?
4. At this point, what questions about this unit would you like to ask your instructor and why?
5. At which stage of the learning process are you in with respect to any topic that you are currently learning? Are you ready to move to the next stage, or should you still work on the ones you have experienced so far?

Short essays

1. Attending classes is rightly considered a very important learning activity. For which of the stages of the learning process is it useful to attend class? What does this tell you about the role of this common learning activity? You may want to discuss separately different teaching styles and class activities that instructors use.
2. Referring to your textbook and your class notes is a common learning activity. For which stages is this useful and for which stages is it not?
3. Most textbooks offer almost exclusively drill type questions. In light of the learning process model, why is this not a good practice? What should they offer instead?
4. Some instructors provide their students with summaries of important formulae. For what stages are these useful? Why would it be more useful to have the students prepare them?
5. Some instructors allow their students to take a small formula (or summary) sheet to the tests. In what way is this beneficial to the students' learning process?
6. Some courses include a "lab" component during which students are to practice, in a more or less structured format, the techniques that have been presented in class. Which stages fit with these lab hours and which do not?
7. What has this unit revealed to you about your learning habits and methods?
8. Given what you have learned from this unit, how do you plan to change your learning habits and methods?
9. Which of the ideas discussed in this unit had you already been using and which one were totally new to you?
10. Researchers, even great and famous ones, are trying to learn something they do not know yet. Describe how the learning process model presented here applies to them: how does each stage enter into their work and what do you think they do in order to move through that stage to the next one?
11. In your opinion, if you start using this learning process model, will learning take more or less time than by using your current method of learning? This is not a yes/no question!

Application questions:

1. It is two days before the start of your new course and you have just bought the textbook. Is it a good idea to browse through the textbook? What goal would you achieve by doing that?
2. It is still two days before the start of the course, you have read the first chapter, have done a the first few exercises, but got an incorrect answer. Should you worry? Provide an explanation for your answer.
3. You are in class, your instructor is lecturing and you are taking good notes. At what stage are you?
4. You are in class, your instructor has proposed an example on the board and gives you some time to try it, which you do. At what stage are you?
5. You are in class, your instructor has proposed an example on the board and given you some time to try it, which you do not do. At what stage are you?
6. The class is now over, you have a one-hour break and decide to go over the notes you took and edit them. At what stage are you?
7. You are now home and try some of the exercises from the book again. And again, you have difficulties with them. At what stage are you and what should you do next?
8. After a class in which your instructor has introduced many new jargon words, you decide to organize them in your personal course glossary. This activity straddles across three stages: which ones?
9. You have some questions to ask to your instructor, but you will not be able to see her for at least five days. What can you do to avoid being bogged down while waiting?
10. Your next major test is in two weeks and you realize that you can't remember an important formula. What do you do and why?
11. Your next major test is in two days and you realize that you can't remember an important formula. What do you do and why?
12. Your next major test is in two weeks and you realize that you don't understand an important method. What do you do and why?
13. Your next major test is in two days and you realize that you don't understand an important method. What do you do and why?

