*Chapter 4: Basic differentiation rules* *Section 1*

# What a differentiation rule is

| What you need to know already: | What you can learn here: |
|---|---|
| ➤ What a derivative is. | ➤ What a rule of differentiation is. |

The only point of this section is to clarify a very common, but major misconception. The fact that a whole section is devoted to this clarification should tell you how important it is; the absence of practice questions tells you that it is a theoretical point, but one that you should always keep in mind in what follows.

In *Chapter 3* we saw that the derivative of a function is given by the limit formula $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. This formula constitutes what a derivative ***is***, what the word *derivative* refers whenever it is used. But we have also seen that computing this limit can be very long and complicated, even for relatively simple functions. The alternative formula $f'(x)=\lim_{t\to x}\frac{f(t)-f(x)}{t-x}$ can make the computations marginally simpler in a few cases, but does not really help much. So, if we want to make any practical use of derivatives we need a more efficient way to compute them. That is what differentiation rules are.

## Definition

A ***differentiation rule*** is a fast and efficient way to compute the derivative of a specific type of functions without using the limit definition and by using as ***few and simple steps*** as possible.

## Warning bells

Contrary to a common, but mistaken belief, a differentiation rule is just a ***short cut*** to compute a derivative, NOT what a derivative is in its essence.

***Example:*** $\left(x^n\right)'=nx^{n-1}$

This derivative formula is correct and you are most likely familiar with it from high school. BUT… it is not what we *mean* by the derivative of a power function! Rather it is what such derivative turns out to be after we compute it. It is what we call the *power rule* of differentiation and in order to use it we must first use the definition to make sure that it is correct!

*Definition, short cut: what's the difference, as long as we do it right?*

The problem is that by ignoring this difference many students end up creating their own rules/definitions, thus making big mistakes in the computation of derivatives. If a procedure to compute a derivative seems simple and reasonable, you may be tempted to use it, but… BEWARE!

## Warning bells

The only rules of differentiations that can be used are those that can be ***proved mathematically*** to be correct.

The fact that something looks like a differentiation rule and looks simple ***does not mean*** that it is correct!

***Example:*** $\left(f(x)\times g(x)\right)'$

How can we compute the derivative of the product of two functions? Even the great Leibniz initially rushed and made the mistake of assuming that such derivative is the product of the corresponding derivatives:

$$\left(f(x)\times g(x)\right)' = f'(x)\times g'(x)$$

Simple and reasonable, but ***WRONG***! We'll soon see what the proper rule is, although you likely know it already.

I hope I have made the point sufficiently clear. From now on, explore all differentiation rules, even find your own, but always check that they are correct before using them and never assume that they constitute a definition.

## Summary

- A differentiation rule is only a short cut to compute a derivative without using the limit definition.

## Common errors to avoid

- Do not think of a differentiation rule as *defining* a derivative.
- Do not use a differentiation rule unless you know that it is correct.

## Learning question for Section D 4-1

1. What is a differentiation rule?

## What questions do you have for your instructor?