Chapter 5: Derivatives of transcendental functions — **Section 3**

Derivatives of Hyperbolic functions

What you need to know already:	*What you can learn here:*
➤ Basic rules of differentiation, including the natural exponential rule.	➤ How to differentiate functions involving hyperbolic functions.

I hope you will not be surprised if I tell you that this section is rather short, since the derivatives of hyperbolic functions are few and easy to find.

I am not surprised, but I am delighted! Let's see them.

Technical fact

The derivatives of the basic hyperbolic functions are as follows:

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$(\tanh x)' = \operatorname{sech}^2 x$$

Proof

Oh, I will not deny you the pleasure of constructing those proofs by yourself. After all, you only need the differentiation rules you have seen so far ☺.

So, off you go to the *Learning questions*!

Summary

➤ The derivatives of hyperbolic functions can be easily obtained by using their defining formulae and the basic rules of differentiation.

Common errors to avoid

➤ Although the differentiation rules for hyperbolic functions are similar to those of trigonometric functions, they are not exactly the same: do not confuse them!

Learning questions for Section D 5-3

Review questions:

1. Explain why the derivatives of hyperbolic functions are so easy to obtain.

Memory questions:

1. What is the derivative of $y = \sinh x$?

2. What is the derivative of $y = \cosh x$?

3. What is the derivative of $y = \tanh x$?

Computation questions:

Compute the derivative of the functions presented in questions 1-22.

1. $y=\cosh(\sinh(x))$
2. $y=\dfrac{\sinh x}{\tanh x}$
3. $y=(\cosh^2 x)\sinh x$
4. $y=\cosh^2(\sinh^2(x))$
5. $f(x)=3^{\cosh(x^2)+2x}$.
6. $y=\cosh(e^x+\ln x)$
7. $y=\dfrac{\sinh(\ln x)}{\cosh^2(x^2+1)}$
8. $y=\dfrac{e^x}{e^2}+\dfrac{88}{x^{72}}-\tanh(3x)$
9. $y=e^x\cosh x$.
10. $y=\tanh\left(\ln\sqrt{\dfrac{1+x}{1-x}}\right)$
11. $f(x)=\log_8\dfrac{e^{2x}+x}{x^2-\sinh x}$.
12. $f(x)=\left(2\sqrt{x^{21}}-\tanh x\right)\sin e-\dfrac{6}{x^3}+4^x$
13. $y=e^{3x^2}\sinh^3 x+4\tanh 2x$
14. $y=\dfrac{\sinh x-\cosh x}{\sinh x+\cosh x}$
15. $y=4x^2\cosh^3 x+2\coth 4x$
16. $f(x)=\dfrac{2\sqrt[3]{x}+\ln x-\tanh x}{8}$
17. $f(x)=\log_3\left(x^2e^{x^2}\right)+\cosh(x^2+4)$
18. $f(x)=\dfrac{2-e^{\sqrt{x}}x}{\cosh\sqrt{x}}$
19. $f(x)=\sinh(\cosh 3^x)$
20. $f(x)=(\sinh x^2)^{x^2}$
21. $f(x)=\dfrac{x^9}{\cosh^8(1-e^x)}$
22. $y=\dfrac{\cosh^8(1-e^x)}{x^9}$

23. Determine the derivative of the function $y=\dfrac{\cosh x^2}{e^{x^2}}$ in the following two ways:

a) By applying appropriate rules to the function as is and simplifying the result algebraically (that is, no need to change the hyperbolic functions).
b) By rewriting the function in terms of a single exponential and computing the derivative of the resulting form.

24. Find *dy/dx* if $x\cosh y = y + x$

Theory questions:

1. What is the 100th derivative of $f(x) = \cosh 2x$?
2. What method is used to obtain the derivative of the two basic hyperbolic functions?
3. What is the derivative of $y = \cosh x \operatorname{sech} x$?

Proof questions:

1. Prove that $(\sinh x)' = \cosh x$
2. Prove that $(\cosh x)' = \sinh x$
3. Prove that $(\tanh x)' = \operatorname{sech}^2 x$
4. Compute the derivative of the other three main hyperbolic functions: $y = \operatorname{sech} x,\ y = \operatorname{csch} x,\ y = \coth x$ starting from their formulae in terms of exponential functions.
5. Compute the derivative of the other three main hyperbolic functions: $y = \operatorname{sech} x,\ y = \operatorname{csch} x,\ y = \coth x$ starting from their formulae in terms of hyperbolic sine and cosine.

Application questions:

For each of the curves presented in question 1-4, determine the equation of the line tangent to it at the given point.

1. $f(x) = e^{\ln(x^2+1)+\cosh x}$ at its *y*-intercept
2. $y = \ln x^2 - \sinh x$ at $(1, -\sinh 1)$.
3. $f(x) = \dfrac{\cosh x}{\sinh x - \cosh x}$ at its *y*-intercept.
4. $x\sinh(x) + y\cos y = \ln 4$ at the point of coordinates $(\ln 2, \ln 2)$.

5. The instantaneous rate of fuel consumption of a car (in the appropriate units) is given by the function $c(v,a) = 2 + \sinh v + \cosh a$, where *v* is the car's velocity and *a* is its acceleration. If the car's velocity is given by $v = \dfrac{t^2}{1+t^2}$.

1) Which function of *t* represents the car's rate of fuel consumption, and is this function differentiable?
2) What is the limiting consumption of the car as *t* increases?

Templated questions:

1. Construct a function involving hyperbolic functions, determine its derivative, its tangent line at some point and its higher derivatives, as much as reasonably possible.

What questions do you have for your instructor?