

## What a word problem is

### What you need to know already:

- ▶ Not much beyond high school math!

### What you can learn here:

- ▶ The key features that identify a question as a *word problem*.
- ▶ Some key aspects to keep in mind when solving a word problem.
- ▶ A general strategy for tackling a word problem.

### *I hate word problems!*

I am not surprised, since most students at your stage do. But as it happens so often, your hate is probably caused by ignorance: you don't have the necessary skills to solve word problems and therefore become frustrated when dealing with one of them. Moreover, being shown how simple the solution of a word problem is can make you even more frustrated, rather than happy. It is almost like watching a magician's trick: you know they are using a reasonable method, most likely a simple one, and when you are shown the method you wonder why you couldn't figure it out by yourself!

Just like what happens when trying to learn a magician's trick, there are two key aspects to become a good problem solver:

- 1) Learn some basic methods for addressing those problems
- 2) Practice extensively and learn from the experience, including both successes and errors.

### *If at first you don't succeed...*

Well, yes, but if you simply try again, without reflecting on what you did and what you achieved, or not, the efforts may be wasted.

So, let's start by identifying what we mean by a word problem, since if you do NOT recognize one when you see it, it may become impossible for you to solve it!

### *Knot on your finger*

A mathematical *exercise* is a question in which all key *formulae are given* and some specific method must be applied in order to arrive at the solution.

A mathematical *word problem* is a question in which some general information is provided, but some key *formula* needed to arrive at the solution *must be constructed* by using information coming from other areas of knowledge.

Notice that once the needed formulae have been constructed, the word problem is reduced to an exercise and you are back to familiar territory.

*So, is it called a word problem because the information is given in words rather than formulae?*

Usually that is true, but the real key reason that separates an exercise from a word problem is that in a problem some formula you need is NOT given to you and you must build it up, from the words provided, but also from other information that you are expected to have or to find on your own. Here is an example.

### **Example: Falling objects**

The following two questions use the same scenario and ask for the same information, but one is an exercise, while the other is a word problem.

#### **Exercise:**

*The height above ground of a certain falling object is given by the function  $h(t) = 52 + 3t - 4.9t^2$ , where  $t$  is in seconds and  $h$  in metres. At what time will the object hit the ground?*

In this case the key formula, a function, is given to you and all you have to do is realize that you need to find the  $x$  intercept of this function and use the corresponding method.

#### **Word problem:**

*An object is thrown up from a height of 52 metres above ground and at a speed of 3 metres per second. After how many seconds will the object reach the ground?*

This time no formula is given, only bits of information and you need to know something from physics in order to be able to use that information to construct the required formula. If you remember the relevant physics facts, you put them into action right away. If not, you search for them in an appropriate way. Once that is done and you have built the formula, it will turn out to be – surprise, surprise –  $h(t) = 52 + 3t - 4.9t^2$  and you are back into exercise mode.

*Is that why I always thought that once I had the formula I could solve a word problem?*

I think so, but notice that what you are saying is really that you don't know how to solve a word problem until someone else changes it into an exercise for you! But you cannot expect that to happen on a regular basis; instead, you must learn how to construct the formulae that you need *on your own*.

*And how do I do that?*

There is no single golden bullet or strategy that will work, since different problems require different strategies, and also since different people prefer different approaches. However, here is a list of hints that you may want to try in order to begin the development of your experience and of your own strategy.

### **Knot on your finger**

#### **Key tips for solving word problems**

- Ensure that the **meaning of all words** used in the problem is clear to you, even by looking them up in a dictionary. As a word problem, words can be the problem!
- Ensure that **your interpretation of the wording** used in the problem is accurate, or you'll end up using wrong data or solving a different problem.
- Identify all the **quantities that are relevant to the problem**, whether they are explicitly mentioned or not. If a quantity is needed and you miss it, you may not be able to construct any formula.
- Identify any **quantitative relationships** you consider relevant to the problem. Once again, missing this aspect will keep you away from the needed formula.

- Search for the *proper form of the relationship* you have identified, unless you are absolutely sure of your knowledge. Wrong relationship implies wrong formula and incorrect solution.
- Use *variable names and symbols* that remind you of the quantities they represent, or you'll mix them up when working with the formulae.
- Draw a *graphical representation* of the problem whenever this is possible. A visual idea of the setting can go a long way towards helping you identify the needed relationship.

- Ask yourself continually if *what you are doing makes sense*, so that you will not waste time on the wrong track.
- Check that *your work is complete* and that all questions posed in the problem have been answered in the way they are expected.

*Sounds reasonable, but very theoretical. How do I implement all this?*

That is what you will see in the next sections for the two most common and traditional types of word problems requiring calculus. So, there will be no challenging learning questions for you here, but in those later sections you will find lots of word problems on which to practice.

### *Summary*

- A word problem is a mathematical question for which a formula must be constructed before any computational method can be used.
- Developing good skills in solving word problems requires some basic methods, lots of practice and, even more important, a lot of reflection on the strategy you are developing and its effectiveness and weaknesses.

### *Common errors to avoid*

- Don't attempt word problems without a plan and without reflection: it will only increase your level of frustration.

## ***Learning questions for Section D 9-1***

### **Review questions:**

1. Explain what a word problem is and how it is different from an exercise described in words.

### **Memory questions:**

Since one the steps in solving a word problem is recalling a basic formula studied earlier, I am including here some formulae from high school that we expect you to know and be familiar with.

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| <ol style="list-style-type: none"><li>1. What makes a mathematical question a <i>word problem</i>?</li><li>2. Which formula provides the volume of a rectangular box?</li><li>3. Which formula provides the volume of a right circular cone?</li></ol> | <ol style="list-style-type: none"><li>4. Which formula provides the volume of a cylinder?</li><li>5. Which formula provides the volume of a sphere?</li><li>6. Which formula provides the surface area of a sphere?</li></ol> |
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### **Theory questions:**

1. Here is a problem that is well known in the educational community. Try to solve it and reflect on your solution. A ship has 26 sheep and 10 goats. How old is its captain?

### **Templated questions:**

1. Classify each question you see in any mathematical book as an exercise or a word problem. What do you use to arrive at your choice?

***What questions do you have for your instructor?***