*Chapter 1: Geometric vectors* *Section 1*

# Why geometric vectors?

| What you need to know already: | What you can learn here: |
|---|---|
| ➢ Just some basic, general facts of everyday life. | ➢ A very initial motivation and introduction to what geometric vectors are and where they may appear. |

*Aren't vectors those quantities that have both a size and a direction?*

The vectors that we shall analyze in this chapter – and that you have seen before – can indeed be characterized in this way. I am sure that some of the following examples have been proposed to you, but perhaps not all. These examples form only an extremely small set of the many, many applications that vectors have in all aspects of modern society.

***Example:* Driving a car**

When you drive a car, you travel at a certain speed, but also in a certain direction, so that your velocity is determined by size as well a N-S component and an E-W component.

***Example:* Flying a plane**

The total force acting on a flying airplane consists of a downward component (generated by gravity and lift), a forward component (generated by the engine and friction) and a sideways component (generated by wind and craft controls).

***Example:* Academic performance**

Your performance in a course is determined by several marks in a variety of tests and quizzes, which are then combined to arrive at your final grade.

***Example:* Time usage**

The way you use your time during a typical week can be described as a combination of time you spend for studying, for personal care, for social needs, for leisure etc., all adding up to 168 hours per week.

***Example:* Value of a car**

The appeal of a car you want to buy may depend on several items, such as engine power, number of seats, price, fuel consumption, and so on.

***Example:* Wealth assessment**

The wealth of a country, or even a person, consists of several distinct items that are inter-related. For a country these can be natural resources, manufacturing, tourism, education etc. For an individual they may involve cash, stocks, real estate, health, family support etc.

*But some of the examples you gave do not seem to have a length and a direction, just some quantitative features.*

Yes, and that is a key fact to notice. While vectors are usually introduced as being characterized by a length and a direction, they are much more than that.

*So, what would you say that a vector is?*

Great question, especially since it does not have a single answer! Let me start from a basic description (I would not call it a definition), but be prepared for the modifications, refinements, extensions and added technicalities we shall add as we proceed in the course.

### Knots on your finger

Generally speaking a ***vector*** is a quantifiable entity that cannot be completely identified by a single number.

*Pretty vague!*

Yes, I am just starting to point to the main feature. We shall start with the simple case and we'll build a rich and growing structure from there.

*And the simple case is that of geometric vectors?*

Yes, and here is the first refinement.

### Definition

A ***geometric vector*** can be represented visually by an arrow in space, beginning at one point and ending at another.

Of course this is still a fairly vague and informal definition, but one that still makes sense, given what you have seen before. We will analyze the more formal general definitions later.

*OK, I am ready to analyze geometric vectors in more depth.*

Good, and that's what we'll do starting from the next section. But first let me introduce a jargon word that is probably familiar to you, but which is best clarified at the outset, since many of the definitions and properties related to vectors use it.

### Definition

The word ***scalar*** is used in linear algebra to indicate a single real number, generally used in combination with vector definitions and operations.

Usually a scalar is denoted by a lower case italic letter, as in *c*.

***Example:***

If you lift a 10 kg box from the floor, you are applying a force, which is a vector, since it requires both a size and a direction. The number 10, however, is a scalar, since it is a single number which will be used in computations involving that force.

Also, if I say that I can only lift something half as heavy, the value ½ is a scalar, since it is another number related to the analysis of the force involved.

## Summary

- Vectors are used to represent quantitative entities that cannot be properly described by a single number.
- A usual number is often described as a scalar when juxtaposed to a vector.

## Common errors to avoid

- Vectors are not just ways of representing quantities that have a size and a direction, but this elementary application is just one of many uses that can be made of the general concept of vector.

## Learning questions for Section LA 1-1

### Review questions:

1. Describe what a vector is and what a geometric vector is.
2. Explain the difference between a vector and a scalar.

### Memory questions:

1. What is a vector?
2. What is a scalar?
3. How are scalars usually denoted?

### Theory questions:

1. What is the difference between a real number and a scalar?
2. What is the difference between a scalar and a vector?
3. What is the similarity between a scalar and a vector?
4. Why is it important to know how a scalar is usually denoted?
5. What does a date of birth have in common with vectors?

6. What does the phone number of my relatives in Europe have in common with vectors?

## ***Templated questions:***

1. For any situation you encounter in other courses or in everyday life, ask yourself if and how vectors can be used.

## ***What questions do you have for your instructor?***