Chapter 10: Eigenvalues and diagonalization — **Section 0**

Introduction to

Eigenvalues and diagonalization

What you need to know already:	What you can learn here:
➤ All the basic information about vectors, matrices, linear transformations and subspaces. That's all… ☺	➤ The meaning of certain special numbers that plays a big role in understanding the features of a matrix and are used in many practical application (which, alas, we shall not see here!).

In earlier chapters, we have looked at the connections between determinants, matrices and systems. We shall now add one more connection, namely to linear transformations. And this new connection involves, although superficially, another general mathematical idea that you have seen before, but probably paid no attention to.

This idea is commonly known as the *Fixed-Point Problem*. It arises in the context of any procedure that changes an object to another one, such as a function, or even an operation such as multiplication. The problem consists of finding those objects that, in fact, are not changed by such procedure.

For instance, the function $f(x) = x^2$ has two fixed points, namely 0 and 1, since $f(0) = 0$ and $f(1) = 1$. Similarly, multiplication by a number that is not 0 or 1 only has the fixed number 0. And if we look at the procedure we call *differentiation* the function $f(x) = e^x$ is a fixed point, since $\frac{d}{dx}e^x = e^x$.

When it comes to linear transformations, we may ask, given a matrix $\mathbf{A}$, which vectors $\mathbf{x}$ are left unchanged by $T_{\mathbf{A}}$? That is, which vectors are such that $\mathbf{Ax} = \mathbf{x}$? We may, but we won't, because it turns out that a more general question will lead to easier, more complete, more interesting and more useful answers.

And the question is: given a matrix $\mathbf{A}$, which vectors are simply stretched by $T_{\mathbf{A}}$? That is, which vectors are such that $\mathbf{Ax} = k\mathbf{x}$?

What seems like a simple, nerdy question, turns out to open the door to a great wealth of information and applications, most of which, however, will have to wait until your next math courses.

But we need to start by getting familiar with the context and the terminology.

What questions do you have for your instructor?