

Introduction to Vector spaces

What you need to know already:

- ▶ Euclidean spaces.
- ▶ Subspaces and their properties.
- ▶ Pretty much all theoretical concepts we have seen so far!

What you can learn here:

- ▶ How to extend the notion of vector to a wider context and different objects.
- ▶ How to generalize the procedures and properties we have seen in Euclidean spaces to more abstract vector settings.

In this chapter I will ask you to boldly go further in the use of generalizations, though only to places where other people have gone before.

I can hear the Star Trek theme in the background...

Yes: if with matrices you may have felt that you were not in Kansas anymore, here you may feel like you are starting an interplanetary voyage to the outer regions of the mathematical universe.

But, to try and remain with our feet on the ground, here is the main idea.

We have discovered that Euclidean vectors have many properties that allow us to use them in many problems, some of them leading to practical applications. Some of those properties are shared by matrices, some are not. It turns out that some of those properties are also common to other sets of objects, as you shall see soon.

So, mathematician asked, what are the key properties that make vectors what they are, and what other sets of mathematical interest have those same properties? Those that do can be seen as some sort of generalized or abstract vectors, and we can expect them to behave in a way that is similar to what we know and love about Euclidean vectors.

Are you saying that we shall look for other mathematical objects that share the possibilities open to Euclidean vectors?

Exactly! But rather than rambling on about generalities, let me show you how it is done.

Fasten your seat belt and put on your space helmet...

What questions do you have for your instructor?

