

Introduction to *Determinants*

What you need to know already:

- ▶ What a matrix is.
- ▶ How to perform elementary row operations.
- ▶ The basic algebraic operation doable with matrices.

What you can learn in this chapter:

- ▶ A very useful way to summarize a square matrix into a single number.

So far in this course we started with vectors, which are sets of numbers arranged in order, and moved to matrices, which are sets of numbers arranged in order over rows and columns. Theoretically we could push even further and organize sets of numbers over three dimensions or more.

Wow, that would make things very messy!

Indeed, and it turns out that this generalization does have useful applications, but only at more advanced levels of mathematics. So, instead we shall go in the opposite direction, namely we'll try to summarize a matrix to a single number that contains useful information about the matrix itself.

This is a common idea in mathematics: given a complex entity, can we make sense of it by using a simpler object? You have already seen this done in other contexts:

- ▶ A large set of numbers can be summarized through its *mean*
- ▶ The magnitude of a vector contains summary information about it
- ▶ A complicated 2-dimensional geometric shape can be characterized by its area.

And so on. It turns out that this attempt to summarize a matrix pays off quite handsomely and provides lots of information about the matrix and the systems

related to it. Moreover, it also allows us to identify new interesting problems (and their solutions) and more uses for matrices.

It does sound like a useful tool.

It is, so let us explore the idea and see what it is and where it leads.

By the way, determinants were developed independently of matrices, but still to solve linear systems. This first happened a long, long time ago in a place far, far away (China), but connections were slowly made over the last few centuries. Moreover, although it will look strange to you once you learn what determinants are, in the Western world they were invented *before* matrices! But they were also defined in a much more convoluted way than what you will see. Another instance of how mathematics develops, not only through discovering new methods and ideas, but also by refining and simplifying old one.

I have chosen to provide you with an introduction to determinants that does not correspond to their historical development, nor to the way they are traditionally introduced in a course like this, but hopefully in a way that makes more sense to a newcomer like you.

We'll see...

What questions do you have for your instructor?