Chapter 6: Lines, planes and other straight objects | **Section 2**

Equations of planes in $\mathbb{R}^3$

What you need to know already:	What you can learn here:
➤ What vectors and vector operations are. ➤ What linear systems are and how they are related to matrices. ➤ The basic forms of the equation of a line, both algebraic and vectorial.	➤ How to identify a plane in 3D space by using vector equations and their properties. ➤ How to determine the geometric relations among planes by using matrices and systems.

While equations of lines in $\mathbb{R}^2$ are familiar to all students who have completed high school, equations of lines and planes in $\mathbb{R}^3$ are not as well known. And yet, they are very similar from many points of view. In particular, they are extremely similar if we approach them from a linear algebra point of view.

Isn't the general equation of a plane the same as for a line?

Well, yes and no. The general equation of a plane in $\mathbb{R}^3$ is $ax+by+cz+d=0$, so it has the same structure, but more variables. But how do we know that this is the equation of a plane?

Because my teacher told me so?

Not good enough I am afraid, so let us develop this and other equations of a plane. However, since many students are familiar with the general equation and are willing to accept that its graph is straight, let us write it down for future reference, but keeping in mind that we have not yet proved that it is indeed the equation of a plane. Ah, the pickiness of linear algebra! ☺

Definition

The set of points in $\mathbb{R}^3$ that satisfy an equation of the form:

$$ax+by+cz=d$$

forms indeed a plane (to be proved!) and this is called its ***general equation***.

So, just like in 2D, I can expect more than one form for the equation of a plane, right?

Yes, but first we have to clarify what a plane *is*!

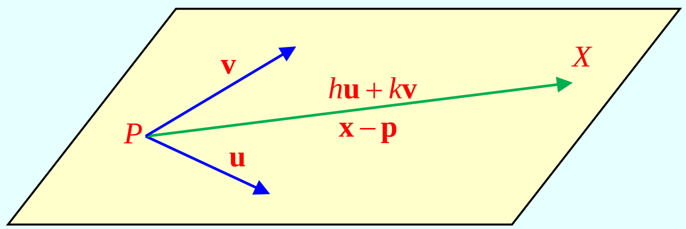
Don't we know that already? It's a straight … plane!

A self- reference will not do. Moreover, once again, what do we mean by *straight*? Following our experience with lines in 2D, we can use an approach based on vectors. And just as we did before, when useful we shall identify a point P with the vector $\mathbf{p}$ whose tail is at the origin and whose tip is at P.

Definition

Given a point $P=(x_0, y_0, z_0)$ in $\mathbb{R}^3$ and two non-parallel vectors $\mathbf{u}=[u_1 \ \ u_2 \ \ u_3]$ and $\mathbf{v}=[v_1 \ \ v_2 \ \ v_3]$, the ***plane*** containing them consists of all points $X=(x, y, z)$ that satisfy the ***vector equation***:

$$\mathbf{x}=\mathbf{p}+h\mathbf{u}+k\mathbf{v}$$



In this case, we shall call the point P a reference point and the vectors $\mathbf{u}$ and $\mathbf{v}$ the ***direction vectors*** of the plane.

Example: $P=(1,1,0)$, $\mathbf{u}=[0 \ \ 2 \ \ 0]$, $\mathbf{v}=[0 \ \ -1 \ \ 1]$

The plane defined by this point and direction vectors consists of all points whose coordinates are such of the form:

$$X=\begin{pmatrix}1\\1\\0\end{pmatrix}+h\begin{bmatrix}0\\2\\0\end{bmatrix}+k\begin{bmatrix}0\\-1\\1\end{bmatrix}=\begin{pmatrix}1\\1+2h-k\\k\end{pmatrix}$$

If we think about these equations, we notice that any such point must have 1 as first coordinate, but can have any other numbers as second and third coordinate. This means that it must be a plane parallel to the y-z plane.

I see: in this way, every other point is in a "straight" plane because the linear combinations of vectors do not allow them to curve away.

That's one way to put it, but notice that you are relying on concepts like "*curve*" and "*straight*" that are not formally clear. However, your intuition is correct: we are using a formal definition to get a handle on a visually clear concept, namely that of a "*straight*" plane. Notice that, as in the case of lines, the vector equation leads naturally to the parametric equations.

Definition

When a vector equation of a plane is expanded by separating its components, it is called a set of ***parametric equations*** of the plane:

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}p_1\\p_2\\p_3\end{bmatrix}+h\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}+k\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}$$

$$\Rightarrow \begin{cases} x=p_1+hu_1+kv_1 \\ y=p_2+hu_2+kv_2 \\ z=p_3+hu_3+kv_3 \end{cases}$$

Here an example will be useful, although I am omitting a visual representation of it, since representing a plane on a flat surface is not always clarifying!

Example: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + h \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + k \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix}$

The plane with this vector equation has parametric equations:

$$\begin{cases} x = 1 - h \\ y = 2 + h + 5k \\ z = 3 + 2h + 4k \end{cases}$$

But there is still another definition that has visual appeal and opens the door to several connections.

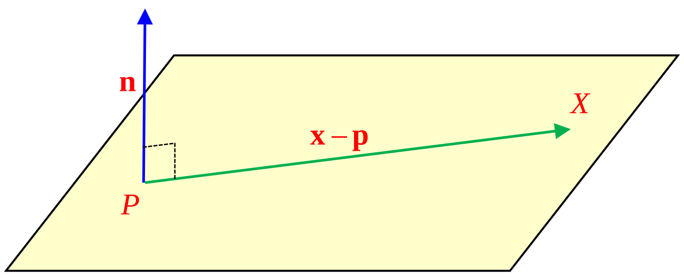
Definition

Given a point $P = (x_0, y_0, z_0)$ and a non-zero vector $\mathbf{n} = [a \;\; b \;\; c]$, the ***plane*** containing P and normal to $\mathbf{n}$ consists of all points X whose coordinates satisfy the ***normal equation***:

$$\mathbf{n} \bullet \mathbf{x} = \mathbf{n} \bullet \mathbf{p} \quad \Leftrightarrow \quad \mathbf{n} \bullet (\mathbf{x} - \mathbf{p}) = 0$$

In this case, we also say that the vector $\mathbf{n}$ is ***normal*** to the plane.

Visually, this is what we are dealing with:



Example: $P = (1, 1, 0), \mathbf{n} = [1 \;\; -1 \;\; 0]$

The plane through this point and with this normal vector consists of all points X whose coordinates are such that:

$$\left(\overline{PX}\right) \bullet \mathbf{n} = 0 \quad \Leftrightarrow \quad (\mathbf{X} - \mathbf{P}) \bullet \mathbf{n} = 0$$

But we can change this equation to a more familiar form:

$$\Leftrightarrow \quad \begin{bmatrix} x-1 \\ y-1 \\ z \end{bmatrix} \bullet \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = 0 \quad \Leftrightarrow \quad x - y = 0$$

This is the general equation of this plane. You can see that any point on this plane can have any value as third coordinate, but its first two must satisfy the remaining condition. This means that it must be the vertical plane above and below to the line $x - y = 0$ in the x-y plane.

The old definition was in terms of two vectors in the plane, while this new definition is in terms of one vector perpendicular to the plane. Do we need to check that they are consistent?

You are definitely getting into the linear algebra spirit! This is exactly our next task and to accomplish it we rely on some vector properties.

Technical fact

Given two ***non-parallel vectors*** **u** and **v** in $\mathbb{R}^3$, there are ***infinitely many*** non-zero vectors that are ***perpendicular*** to both **u** and **v** and they form the solution set of the homogeneous system whose matrix of coefficients is $\begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix}$.

Proof

If $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ is a solution of the homogeneous system, it means that:

$$\begin{cases} u_1x_1 + u_2x_2 + u_3x_3 = 0 \\ v_1x_1 + v_2x_2 + v_3x_3 = 0 \end{cases}$$

But this implies that **x** is perpendicular to both **u** and **v**. On the other hand, if **x** is not a solution, one of the two equations is not true and hence **x** is not perpendicular to one of the two vectors.

This fact gives us a way to determine whether a vector is perpendicular to the direction vectors of a plane.

Strategy for constructing the normal equation of a plane from its vector equation

If $X = P + h\mathbf{u} + k\mathbf{v}$ is the vector equation of a plane in $\mathbb{R}^3$, to construct its normal equation:

- ***Obtain one solution*** **n** of the homogeneous system whose matrix of coefficients is $\begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix}$.
- ***Construct*** the equation $\mathbf{n} \bullet \mathbf{x} = \mathbf{n} \bullet \mathbf{p}$.

Example: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + h\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + k\begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix}$

Consider the plane with this vector equation. We construct the required matrix of coefficients and use it to determine its normal vectors:

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 5 & 4 \end{bmatrix} \Rightarrow \begin{cases} x = y + 2z \\ 5y = -4z \end{cases}$$

To pick a specific one, all we have to do is pick a value for z, say $z = 5$. We now have a normal vector, namely $\begin{cases} x = 6 \\ y = -4 \\ z = 5 \end{cases} \Rightarrow \mathbf{n} = \begin{bmatrix} 6 \\ -4 \\ 5 \end{bmatrix}$. The normal equation is $\begin{bmatrix} 6 & -4 & 5 \end{bmatrix} \bullet \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 6 & -4 & 5 \end{bmatrix} \bullet \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$.

So, we now know that given the vector equation of a plane we can construct a normal equation for it. Although there are infinitely many equations we can construct in this way, depending on which normal vector and reference point we use, they all identify the same plane. Now we'll go backwards.

Technical fact

Given a plane of normal equation $\mathbf{n} \bullet \mathbf{x} = \mathbf{n} \bullet \mathbf{p}$, there are ***infinitely many*** pairs **u** and **v** of non-parallel vectors both ***perpendicular*** to **n**.

Proof

We can consider the equation $\mathbf{n} \bullet \mathbf{x} = 0$ as a homogeneous system of one equation in two variables. Since $\mathbf{n} \neq \mathbf{0}$, this system has rank one and therefore it has infinitely many solutions with two free variables. By picking two pairs of values for the free variables that are not multiples of each other we obtain two non-parallel vectors both perpendicular to **n**. Finally, we notice that there are infinitely many such pairs.

Strategy for constructing the vector equation of a plane from its normal equation

Given a plane in $\mathbb{R}^3$ of normal equation $\mathbf{n} \bullet \mathbf{x} = \mathbf{n} \bullet \mathbf{p}$, to construct its vector equation:

- ***Obtain two non-parallel solutions*** **u** and **v** of the equation $\mathbf{n} \bullet \mathbf{x} = 0$.
- ***Construct*** the equation $X = P + h\mathbf{u} + k\mathbf{v}$.

Example: $[6\ \ -4\ \ 5] \bullet [x\ \ y\ \ z] = [6\ \ -4\ \ 5] \bullet [1\ \ 2\ \ 3]$

Let's go back to this plane, whose normal equation we just constructed and let's pretend that this is all we know about it. We need non-parallel solutions of the equation:

$$6x - 4y + 5z = 0$$

Since any two solutions will do and since to get two solutions we need to pick values for *y* and *z*, we try the easy route by setting $y = 1,\ z = 0$, thus getting $x = \frac{2}{3}$, and then setting $y = 0,\ z = 1$, thus getting $x = -\frac{5}{6}$. The two vectors are therefore:

$$\mathbf{u} = \left[\frac{2}{3}\ \ 1\ \ 0\right],\ \mathbf{v} = \left[-\frac{5}{6}\ \ 0\ \ 1\right]$$

(Check that they are not parallel: one of the *Learning questions* will ask you to show that the above choices work almost always). Since we do not like fractions (well, I don't!) and since it is only the direction of the vectors that is important, we multiply both vectors by 6, thus getting:

$$\mathbf{u}' = [4\ \ 6\ \ 0],\ \mathbf{v}' = [-5\ \ 0\ \ 6]$$

In conclusion, the vector equation of our plane is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + h\begin{bmatrix} 4 \\ 6 \\ 0 \end{bmatrix} + k\begin{bmatrix} -5 \\ 0 \\ 6 \end{bmatrix}$$

Notice that this is not the vector equation from which we started in the earlier example, but it is the same plane. Can you prove that?

So, as you can see, we can go from one form of the equation to the other fairly easily.

Cool, but we seem to have lost the general equation in all this!

Not really: it is right there in front of you, in plain sight!

Technical fact

When a normal equation of a plane is expanded into its components, it provides its ***general equation***.

Proof

Well, let's expand it!

$$\mathbf{n} \bullet \mathbf{x} = \mathbf{n} \bullet \mathbf{p}$$

$$\Rightarrow \quad n_1 x_1 + n_2 x_2 + n_3 x_3 = n_1 p_1 + n_2 p_2 + n_3 p_3$$

But the right side is just one constant, and by changing the letter representing the components of **n**, we can write this as:

$$ax + by + cz = d$$

This is the general form, as claimed.

Example: $\begin{bmatrix} 3 & 1 & -2 \end{bmatrix} \bullet \begin{bmatrix} x-1 & y+2 & z-5 \end{bmatrix} = 0$

The plane with this normal equation has general equation given by:

$$3(x-1) + 1(y+2) - 2(z-5) = 0 \quad \Leftrightarrow \quad 3x + y - 2z = -9$$

And of course, we can go backwards.

Strategy for constructing the normal equation of a plane from its general equation

If $ax + by + cz = d$ is the general equation of a plane in $\mathbb{R}^3$, to construct its normal equation:

- ***Pick*** $\mathbf{n} = \begin{bmatrix} a & b & c \end{bmatrix}$ as normal vector
- ***Pick any point*** **p** by suitably selecting the value of two variables and computing the third.
- ***Construct*** the equation $\mathbf{n} \bullet \mathbf{x} = \mathbf{n} \bullet \mathbf{p}$.

Example: $3x + y - 2z = -9$

Given the plane with this general equation, we can use $\mathbf{n} = \begin{bmatrix} 3 & 1 & -2 \end{bmatrix}$ and the point $\mathbf{P}(-3, 0, 0)$ to arrive at the normal equation:

$$\begin{bmatrix} 3 & 1 & -2 \end{bmatrix} \bullet \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 3 & 1 & -2 \end{bmatrix} \bullet \begin{bmatrix} -3 & 0 & 0 \end{bmatrix}.$$

Wait now: how did you pick the point?

Theoretically we can pick any point whatsoever, but we might as well pick one that is easy to compute. To do that you may want to remember this little trick:

Strategy for selecting a point on a plane of given general equation

If $ax+by+cz=d$ is the general equation of a plane in $\mathbb{R}^3$, to pick an easily computable point on it:

- ***Select*** one variable whose coefficient is ***not 0***.
- If more than one choice is possible, ***select***, if possible, one whose coefficient is a ***divisor of the constant***.
- ***Set*** the value of the other two variables to be ***0***
- ***Compute*** the value of the selected variable to be the solution of the resulting equation.

Notice that this is exactly what I did in an earlier example. This strategy is simple enough that I refer you to the *Learning questions* for further examples.

Notice also that by using the above strategies we can switch between the vector and normal equations, or between normal and general equation. In theory, this allows us also to switch between vector and general equation (just go through the intermediate step of constructing a normal equation). However, there is a more direct way to do this.

Strategy for constructing the vector equation of a plane from its general equation

If $ax+by+cz=d$ is the general equation of a plane in $\mathbb{R}^3$, to construct its vector equation:

- ***Select*** three non-collinear points on the plane, say P, Q and R, by a suitable choice of some of the variables.
- ***Construct*** the vectors $\overline{PQ}$ and $\overline{PR}$, or any other pair of non-parallel vectors based on them.
- ***Pick*** P as reference vector
- ***Construct*** the equation $X=P+h\overline{PQ}+k\overline{PR}$.

Example: $3x+y-2z=-9$

Given this general equation, we can pick the points $P=(-3,0,0)$, $Q=(0,-9,0)$ and $R=(0,0,4.5)$ from which we can construct the vectors:

$$\overline{PQ}=[3\ \ -9\ \ 0],\ \overline{PR}=[3\ \ 0\ \ 4.5]$$

Therefore, the vector equation of the plane is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}+h\begin{bmatrix} 3 \\ -9 \\ 0 \end{bmatrix}+k\begin{bmatrix} 3 \\ 0 \\ 4.5 \end{bmatrix}.$$

This also tells us that the parametric equations are $\begin{cases} x = -3 + 3h + 3k \\ y = -9h \\ z = 4.5k \end{cases}$

Of course, different choices of points would lead to alternative, but equivalent equations.

We can also go from the vector equation to the general equation directly, but this is simple and fun enough that I will leave the corresponding strategy to the *Learning questions*.

It seems that it all fits together! If so, in the previous section we looked at the intersection of several lines. Can we also look at the intersections of several planes?

Certainly, and in fact the options here are more numerous and more interesting than with 2D lines.

Knot on your finger

If $a_i x + b_i y + c_i z = d_i, 1 \le i \le n$ are the general equations of n planes in $\mathbb{R}^3$, their ***points of intersection*** are given by the solutions of the system whose augmented matrix is $\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ \ldots & \ldots & \ldots & \ldots \\ a_n & b_n & c_n & d_n \end{array}\right]$

Technical fact

The set consisting of the points of intersection of n planes in $\mathbb{R}^3$ contains either:

- ***no*** point, or
- a ***single*** point, or
- ***infinitely many*** points forming a line, or a plane.

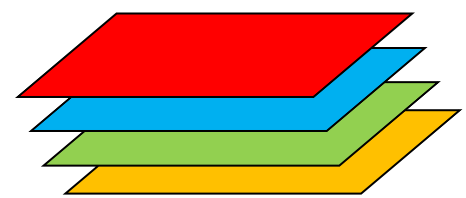
Which of these options occurs depends on the ranks of the matrices involved, as we know about the number of solutions of a system.

Planes that have no common point of intersection can be obtained in three ways. Let's see how these different cases occur.

Example: Parallel planes

If two of the planes are parallel and distinct, they clearly have no point in common, even though some of them may have an intersection. For instance, the planes:

$$\begin{cases} x + 2y - 3z = 1 \\ x + 2y - 3z = 2 \\ 2x + 4y - 6z = 3 \\ -x - 2y + 3z = 1 \end{cases}$$

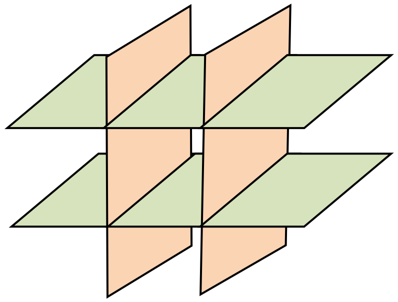


are all parallel and distinct, so that no two of them have anything in common.

However, even if just two of the planes are parallel and different, there is no common point.

For instance these planes:

$$\begin{cases} x+2y-3z=1 \\ x+2y-3z=2 \\ x+y-z=3 \\ x+y-z=1 \end{cases}$$

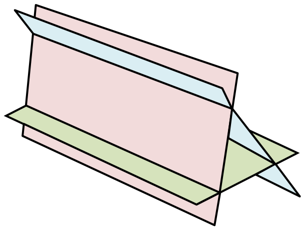


are not all parallel, but the first two are, hence they have no common point, as well as the last two. As a consequence, there is no point that is on ALL four planes. Other similar configurations are possible, but in all cases where two parallel and distinct planes exist, no common point exists.

Example: Planes parallel to a line

The other case occurs when the system consists of three planes, one of which is parallel to the line of intersection of the other two. Here is an example that is easy to visualize:

$$\begin{cases} z=0 \\ 3x-z=0 \\ x+z=1 \end{cases}$$



Here the first two planes intersect in the y axis and the last one is parallel to that axis, so that there are no points common to all planes.

The case where a set of planes contains a single point is rather general and has no specific particular configurations, except for the common point. For instance, the *x*-*y*, *y*-*z* and *z*-*x* planes have a single point in common, namely the origin. But there are many many other possibilities.

Example: $\begin{cases} x+2y+z=3 \\ x+2y+2z=2 \\ 2x-4y+3z=0 \end{cases}$

This system has a unique solution (check it!). That means that the three planes represented by its three equations have a single point in common (find its coordinates!).

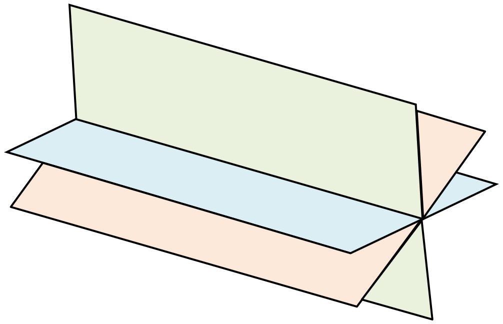
As for infinitely many solutions (points of intersections), one obvious situation is when all equations represent the same plane, as in:

$$\begin{cases} x+2y+z=3 \\ 2x+4y+2z=6 \\ -x-2y-z=-3 \\ 10x+20y+10z=30 \end{cases}$$

But we also have the situation where all planes contain the same line.

Example: $\begin{cases} x+2y+z=3 \\ x+2y+2z=2 \\ 2x+4y+3z=5 \end{cases}$

This system has infinitely many solutions (check it!) and one free variable. That means that the three planes all contain a single line, as shown in the picture. In the next section we shall look at equations of lines in $\mathbb{R}^3$ and we'll be able to identify such line more clearly.



Summary

- Just for lines in $\mathbb{R}^2$, planes in $\mathbb{R}^3$ can be identified by several types of equations by using properties of their points interpreted as vectors.
- Linear systems allow us to determine the intersection set of several planes. Therefore, such set can only include one or infinitely many points, or none at all.

Common errors to avoid

- Work at distinguishing the different types of equations for a plane.
- Work at connecting the different types of equations for a plane.

Learning questions for Section LA 6-2

Review questions:

1. Describe the different types of equations for a plane in $\mathbb{R}^3$.
2. Explain how to construct a type of equation for a plane by starting from a different type.
3. Identify the different ways in which a set of planes can intersect and what the intersection set looks like in each such case.

Memory questions:

1. What is the form of the general equation of a plane?
2. What is the form of the vector equation of a plane?
3. What is the form of the normal equation of a plane?
4. What is the form of the parametric equation of a plane?

Computation questions:

In questions 1-4 you are given the general equation of a plane. Use it to construct the normal, vector and parametric equations of that plane.

1. $3x+2y-z=4$
2. $3x-5y+2z=4$
3. $x+y=1$
4. $z=3x-2y+1$

In questions 5-14 you are given some information about a plane. Use it to construct the general, vector, normal and parametric equations of that plane.

5. Contains the point (1, 2, 3) and the vectors [-1 1 2] and [0 5 4].
6. Contains the point (1, 2, 3) and the vectors [1 1 1] and [2 3 4].
7. Contains the point (1, 2, 3) and the vectors $\begin{bmatrix}4 & 5 & 4\end{bmatrix}$ and $\begin{bmatrix}-1 & -2 & 0\end{bmatrix}$.
8. Contains the points (2, 1, 0) and (3, 4, 5), and the vector [1 2 3] .
9. Contains the point (4, 0, -5) and is perpendicular to the vector $\mathbf{v}=\begin{bmatrix}3 & 1 & -6\end{bmatrix}$
10. Contains the points (1, 1, 1), and is perpendicular to the vector [4 0 2].
11. Contains the point $P=(3,-5,2)$ and is orthogonal to the line containing the same point P and $Q=(1,1,1)$.
12. Contains the points (1, 1, 1) and (2, 3, 4) and the vector [4 0 2].
13. Contains the three points of coordinates (1,1,1), (4,0,2) and (0,1,-1)
14. Contains the points $(2,-6,3),(-1,5,2),(1,1,1)$.

In questions 15-17 determine the set of points that are contained in all the given planes.

15. $\begin{cases} x-y+2z=13 \\ 6x-2y+4z=5 \\ x+y-z=2 \end{cases}$

16. $\begin{cases} 2x+3y+z=25 \\ -x-2y+4z=-25 \\ 3x-y+2z=-2 \end{cases}$

17. $\begin{cases} 3x-y+2z=1 \\ 6x-2y+4z=2 \\ x+y-z=2 \end{cases}$

Theory questions:

1. Which type of equation is equivalent to the normal equation, just in different form?
2. How do we use the parametric form of the equation of a plane to obtain its vector form?

3. How are the parametric equations of a plane obtained from its vector equation?

4. How many points are needed to identify a plane?

5. What does each equation of a homogeneous linear equation represent geometrically?

6. What does the solution set of a homogeneous system represent in $\mathbb{R}^3$, geometrically?

7. If the general equations of three planes in $\mathbb{R}^3$ form a consistent system whose augmented matrix has rank 2, what geometric set is formed by their intersection?

8. Which form of the equation of a plane is obtained by writing the vector equation component by component?

9. Does the vector equation of a plane express the variable point as a linear combination of other vectors?

Proof questions:

1. Show that any vector joining two points on the plane $\mathbf{X} = \mathbf{P} + h\mathbf{u} + k\mathbf{v}$ is a linear combination of **u** and **v**.

2. Prove that any plane in $\mathbb{R}^3$ has infinitely many different vector, parametric and general equations.

3. Show that any vector perpendicular to **u** and **v** is perpendicular to any vector in the plane $\mathbf{X} = \mathbf{P} + h\mathbf{u} + k\mathbf{v}$.

4. Prove that the distance from $\mathbf{p} = \begin{bmatrix} x_0 & y_0 & z_0 \end{bmatrix}$ to $ax + by + cz + d = 0$ is $d = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$

Templated questions:

1. Construct a general equation of a plane and from it obtain the vector and normal equations of the same plane.

2. Construct a vector equation of a plane and from it obtain the normal and general equations of the same plane.

3. Construct a normal equation of a plane and from it obtain the vector and general equations of the same plane.

4. Choose three separate points in $\mathbb{R}^3$ and construct an equation for the plane that contains them.

What questions do you have for your instructor?