# Basic Algebra Formulae

### Fractions

$$\frac{a+b}{d}=\frac{a}{d}+\frac{b}{d}$$

$$\frac{a}{c}+\frac{b}{d}=\frac{ad+bc}{cd}$$

$$\frac{a}{c}\bullet\frac{b}{d}=\frac{ab}{cd}$$

$$\frac{a}{c}\div\frac{b}{d}=\frac{ad}{cb}$$

### Factoring/Expanding

$$a^2-b^2=(a+b)(a-b)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^4+b^4=(a^2+\sqrt{2}ab+b^2)(a^2-\sqrt{2}ab+b^2)$$

$$(a+b)^2=a^2+2ab+b^2$$

$$(a+b)^3=a^3+3a^2b+3ab^2+b^3$$

$$n!=1\bullet 2\bullet \ldots \bullet n$$

$$0!=1$$

### Exponents

$$a^n a^m=a^{n+m}$$

$$\frac{a^n}{a^m}=a^{n-m}$$

$$a^{-n}=\frac{1}{a^n}$$

$$\left(a^n\right)^m=a^{nm}$$

$$a^{\frac{n}{d}}=\sqrt[d]{a^n}$$

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n$$

### Logarithms

For $a, b, x, y>0$:

$$c=\log_b x \iff x=b^c$$

$$\log_b(xy)=\log_b x+\log_b y$$

$$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$$

$$\log_b x^n=n\log_b x$$

$$\log_b x=\frac{\log_a x}{\log_a b}$$

### Quadratic formula

The solutions of $ax^2+bx+c=0$ are:

$$x=\frac{-b\mp\sqrt{b^2-4ac}}{2a}$$

### Biquadratic formula

The solutions of $ax^4+bx^2+c=0$ are:

$$x=\mp\sqrt{\frac{-b\mp\sqrt{b^2-4ac}}{2a}}$$

### Completing the square

$$ax^2+bx+c \quad = \quad a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$$

# Basic Analytic Geometry Formulae

### Distance formula

From $(x_1, y_1)$ to $(x_2, y_2)$:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

### Midpoint formula

Between $(x_1, y_1)$ and $(x_2, y_2)$:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

### Intercepts

Given $y=f(x)$, to find

*x-intercepts:* *Set* $y=0$

*y-intercept:* *Set* $x=0$

### Equations for a line

*General:* $ax+by+c=0$

*Slope-intercept:* $y=mx+b$

*Point-slope:* $y=m(x-x_0)+y_0$

*Slope between* $(x_1, y_1)$ *and* $(x_2, y_2)$:

$$m=\frac{rise}{run}=\frac{y_2-y_1}{x_2-x_1}$$

### Equations of conics

*Parabola:* $y=ax^2+bx+c$

*Ellipse:* $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

*Circle:* $x^2+y^2=r^2$

*Hyperbola:* $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

### Other basic functions

*Cubic:* $y=ax^3+bx^2+cx+d$

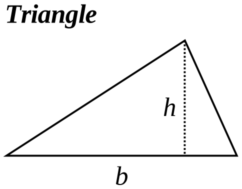
*Polynomial:* $y=a_0+a_1x+a_2x^2+\cdots+a_nx^n$

*Rational:* $y=\frac{a_0+a_1x+a_2x^2+\cdots+a_nx^n}{b_0+b_1x+b_2x^2+\cdots+b_mx^m}$

# Basic Plane Geometry Formulae

### Triangle

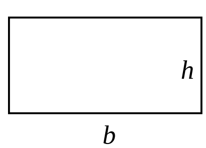
$$A=\frac{bh}{2}$$




### Rectangle

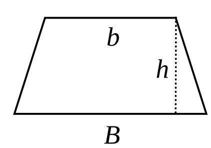
$$A=bh$$

$$P=2b+2h$$




### Trapezoid

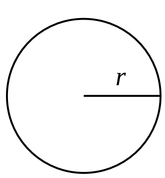
$$A=\frac{(B+b)h}{2}$$




### Circle

$$C=2\pi r$$

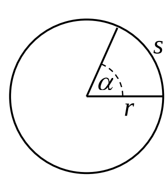
$$A=\pi r^2$$




### Circular sector

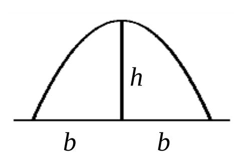
$$s=\alpha r$$

$$A=\frac{\alpha r^2}{2}$$

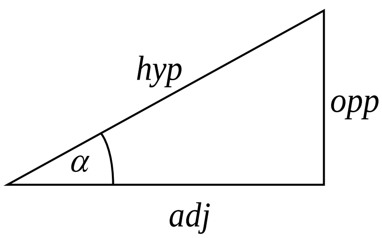



### Parabolic cap

$$A=\frac{bh}{3}$$



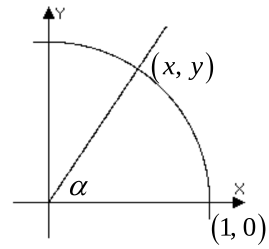
# Basic Trigonometry formulae



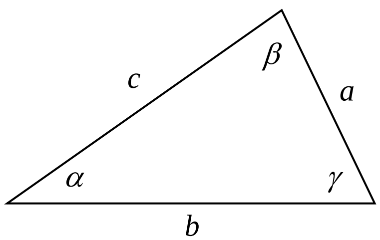

$$\sin\alpha = \frac{opp}{hyp} = y$$

$$\cos\alpha = \frac{adj}{hyp} = x$$

$$\tan\alpha = \frac{opp}{adj} = \frac{y}{x}$$



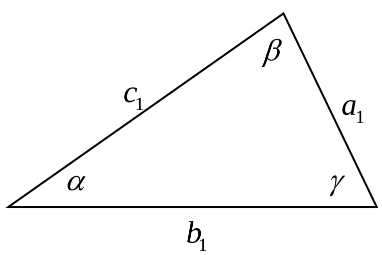

| Reciprocal identities | Pythagorean identities | Double angle formulae | Addition formulae | Cofunction identities |
|---|---|---|---|---|
| $\sec\alpha = \frac{1}{\cos\alpha}$ | $\sin^2\alpha + \cos^2\alpha = 1$ | $\sin 2\alpha = 2\sin\alpha\cos\alpha$ | $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ | $\cos\alpha = \sin\left(\frac{\pi}{2} - \alpha\right)$ |
| $\csc\alpha = \frac{1}{\sin\alpha}$ | $\sec^2\alpha = \tan^2\alpha + 1$ | $\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ | $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ | $\cot\alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$ |
| $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ | $\csc^2\alpha = \cot^2\alpha + 1$ | $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$ | $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$ | $\csc\alpha = \sec\left(\frac{\pi}{2} - \alpha\right)$ |

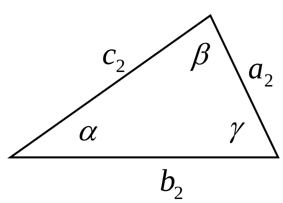



Law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

# Similar triangles



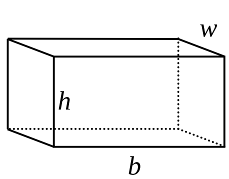




$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

# Basic Solid Geometry Formulae

### Box

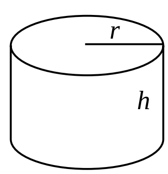
$V = bhw$




### Cylinder

$V = \pi r^2 h$

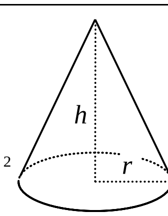
$A = 2\pi rh + 2\pi r^2$




### Cone

$V = \frac{\pi r^2 h}{3}$

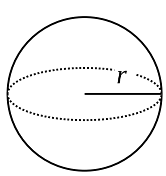
$A = \pi r\sqrt{r^2 + h^2} + \pi r^2$




### Sphere

$V = \frac{4\pi r^3}{3}$

$S = 4\pi r^2$

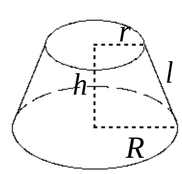



### Frustum of a cone

$V = \frac{\pi\left(R^2 - r^2\right)h}{3}$

*Slant area:*

$= \pi\left(R + r\right)l$



# *Basic Differentiation Formulae*

In the following, $u = f(x)$ and $v = g(x)$ are functions, while $a$, $b$, $c$ and $n$ are real constants.

| | | | |
|---|---|---|---|
| *Additive constant rule:* | $\frac{d}{dx}[c] = 0$ | Power rule: | $\frac{d}{dx}[x^n] = nx^{n-1}$ |
| *Multiplicative constant rule:* | $\frac{d}{dx}[cu] = cu'$ | Product rule: | $\frac{d}{dx}[uv] = u'v + uv'$ |
| Linear rule: | $\frac{d}{dx}[ax+b] = a$ | Quotient rule: | $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$ |
| Addition rule: | $\frac{d}{dx}[u \pm v] = u' \pm v'$ | Chain rule: | $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$ |

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$

$$\frac{d}{dx}[\operatorname{arccot} x] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[a^x] = a^x \ln a$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$$

# *Basic Integration Formulae*

$$\int af(x)dx = a\int f(x)dx$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int dx = x + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$$

$$\int x^{-1}dx = \int \frac{1}{x}dx = \ln|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \sin x\, dx = -\cos x + c$$

$$\int \cos x\, dx = \sin x + c$$

$$\int \sec x \tan x\, dx = \sec x + c$$

$$\int \csc x \cot x\, dx = -\csc x + c$$

$$\int \tan x\, dx = -\ln|\cos x| + c$$

$$\int \cot x\, dx = \ln|\sin x| + c$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + c$$

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + c$$

$$\int \sec^2 x\, dx = \tan x + c$$

$$\int \csc^2 x\, dx = -\cot x + c$$

$$\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \arcsin\frac{x}{a} + c$$